

MATH ACTIVITY BOOK

Grades
5-8+

Applying Pre-Algebra

Reproducible Problems & Activities • Everyday Use of Concepts • Number Systems • Integers
• Inverse & Identities • Radicals, Square, & Cube Roots • Order of Operations • Variables
& Exponents • Linear Equations • Rectangular Coordinates • Graphing

Watts = amps x volts



Volume = length x width x height

By Robert A. Sadler, Ph. D.

Speed = frequency x wavelength

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Applying Pre-Algebra

By

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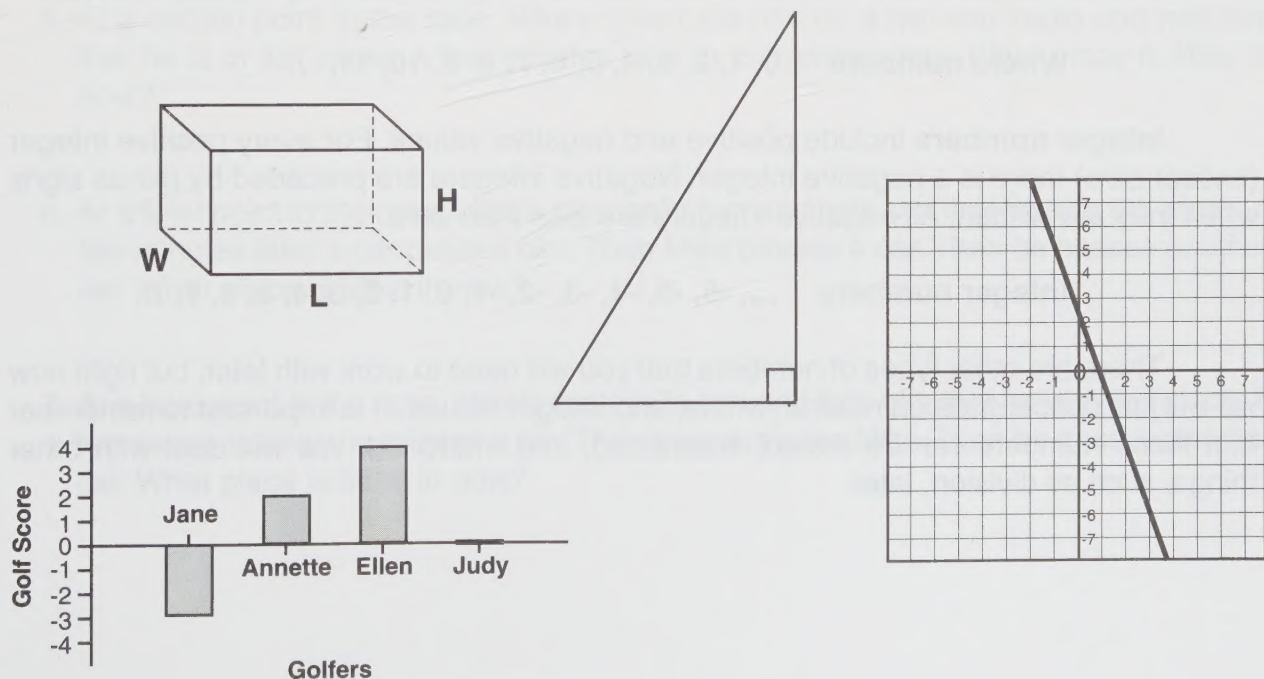
INTRODUCTION

Pre-algebra is a crucial component of a student's mathematical development. It introduces or reinforces many of the basic mathematical concepts that a student will need to apply in succeeding courses. It treats these concepts in a concrete manner that is, perhaps, friendlier than the abstractions that they will encounter in geometry, algebra, trigonometry, or pre-calculus. By necessity, most pre-algebra texts introduce the requisite topics and provide exercises that highlight the concepts and reinforce the basic manipulative skills.

This book is different. It assumes that a student has had an introduction to pre-algebra and is designed to provide practical problems that exercise the skills that the student has acquired. The emphasis here is not on new material or on drill-and-practice exercises that refine manipulative skills. Instead, almost all of the problems are "word problems," ones that imbed quantitative problems in verbal scenarios.

Problems are different from exercises. The latter usually require the application of routine procedures to process an arithmetic expression into a value that is the solution. Quantitative problems are more demanding and usually activate more cognitive skills than exercises. They demand that a student extract relevant information from a verbal or pictorial scenario, devise a plan to arrive at a solution, set up initial arithmetic expressions that describe the problem, perform the necessary operations to arrive at a solution, and then evaluate the solution so that it is satisfactory with respect to both mathematics and common sense.

Teachers are encouraged to copy the pages of this book for use in their classrooms. It is hoped that the problems therein will contribute to the development of lifelong problem-solving skills on the part of the students and suggest some of the elegance and intellectual satisfaction that the author finds in mathematics.



Name _____ Date _____

LESSON ONE: NUMBER PROBLEMS

We use numbers for many different things in our lives. It is tempting to say that numbers are just numbers and that they are all basically alike. If we look closer, however, we see that there are several number systems whose properties are different from one another.

In one scheme, numbers can be classified as **cardinal numbers**, **ordinal numbers**, or **nominal numbers**. **Cardinal numbers** are numbers that are used to indicate how many things are in a group, **ordinal numbers** are numbers that indicate the order of objects in a group, and **nominal numbers** are used to identify a particular person, place, or thing.

The number of players on the court in a basketball game or the number of eggs in a dozen are examples of cardinal numbers.

The order in which teams or persons finish in a contest is described by ordinal numbers. Often they are expressed with st, nd, rd, or th endings. Examples include 1st, 2nd, 3rd, or 5th.

The number address on a house, the number on a policeman's badge, or the number on a baseball player's uniform are examples of nominal numbers.

Another way to classify numbers is as **natural numbers** (sometimes called counting numbers), **whole numbers**, or **integer numbers**.

Natural numbers are the numbers that we learned to count with. They begin with 1 and continue up to arbitrarily large numbers. They never contain decimal places or fractional parts.

Natural numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...

Whole numbers are like the natural numbers except that they include zero (0).

Whole numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...

Integer numbers include positive and negative values. For every positive integer (except zero) there is a negative integer. Negative integers are preceded by minus signs when they are written. All negative integers are less than zero.

Integer numbers ..., -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, ...

There are other types of numbers that you will need to work with later, but right now we will just concentrate on natural, whole, and integer values. It is important to remember that these numbers can be added, subtracted, and multiplied. We will deal with other things, such as division, later.

Name _____ Date _____

LESSON ONE: Exercise 1

The exercises below all use natural numbers, whole numbers, or integer numbers.

Mary works on Saturdays with her mother in a craft shop in Montana. All items for sale in the shop are sold for \$1, \$3, or \$5, and Montana has no sales tax. Mary works the cash register and totals up the sales for customers.

1. A customer buys 4 \$1 items, 6 \$3 items, and 2 \$5 items. How much should Mary charge her?
2. A customer buys 1 \$1 item, 2 \$3 items, and 5 \$5 items. How much should Mary charge her?
3. A customer buys 3 \$1 items, 9 \$3 items, and 3 \$5 items. How much should Mary charge him?
4. A customer buys 21 \$1 items, 12 \$3 items, and 10 \$5 items. How much should Mary charge her?

Mike Motor is racing in a road race that extends along a long road in the desert.

5. At a certain point in the race, Mike's crew calls him on a two-way radio and tells him that he is in 4th place. A few minutes later, a car passes him. What place is Mike in now?
6. At a later point in the race, Mike's crew calls him and tells him that he is in 5th place. A few minutes later a car passes him. Then Mike passes a car. Then he passes another car. What place is Mike in now?
7. At a later point in the race, Mike's crew calls him and tells him that he is in 3rd place. A few seconds later, he passes a car. Then two cars pass Mike. Then he passes another car. What place is Mike in now?

Name _____ Date _____

LESSON ONE: Exercise 2

Jill lives in a tall apartment building in a Midwestern city. Sometimes she works out by running up and down the stairways.

1. Jill starts running on the 7th floor. Then, she runs up two floors of stairs, then down three floors of stairs, and then up five floors of stairs. On what floor does she end up?

2. Jill starts running on the 7th floor. Then, she runs up seven floors of stairs, then down four floors of stairs, then up 12 floors of stairs, and then down five floors of stairs. On what floor does she end up?

3. Jill starts running on the 21st floor. Then, she runs up 20 floors of stairs, then down five floors of stairs, then up 10 floors, then down three floors, and then up another 15 floors. On what floor does she end up?

4. Jill starts running on the 34th floor. Then, she runs up five floors of stairs, then down two floors of stairs, and then up seven floors of stairs. Then, she runs up 11 floors of stairs, down five floors, up six floors, and (finally) down nine floors. On what floor does she end up?

Name _____ Date _____

LESSON ONE: Exercise 3

A highway in a large Western state is straight as an arrow and runs in an east-west direction. There are mile markers set one mile apart on the highway. They are numbered beginning with mile 1, one mile from the eastern border of the state, to mile 300, on the western border.

A state policeman patrols a certain stretch of the road and often needs to know where he is. Sometimes he remembers a mile marker, but at other times he just remembers how far he has traveled east or west.

1. The policeman starts at mile marker 44 and travels 21 miles east to a traffic jam. He then travels seven more miles east where he stops to pick up litter along the road. He then drives 17 miles west to look for a lost dog. He then drives four miles east to a stalled car. At what mile marker was the stalled car?
2. The policeman starts at mile marker 10 and travels eight miles east to a disturbance. He then travels 13 miles west where he puts up a road construction sign. He then drives nine miles east to look for a runaway child. He then drives seven miles west and finds an abandoned truck. At what mile marker was the abandoned truck?
3. The policeman starts at mile marker 21 and travels 25 miles west to a stalled vehicle. He then travels 10 miles east where he stops a motorist for speeding and gives her a ticket. He then drives 11 miles west to check out a disturbance at the edge of the road. He then drives 14 miles east to a spot where a car has run out of gas. Finally, he drives nine more miles east to a car accident. At what mile marker was the accident?
4. The policeman starts at mile marker 50 and travels five miles west, where he arrests a suspected drug trafficker. He then travels 18 miles east where he helps firemen put out a fire in a truck carrying hazardous chemicals. He then drives 21 miles west to a disturbance in a camper trailer parked at the edge of the road. He then drives 15 miles east to a spot where he arrests a bank robber. Finally, he drives eight miles west chasing a speeder before pulling him over. At what mile marker does he pull over the speeder?

Name _____ Date _____

LESSON ONE: Exercise 4

Air temperature is sometimes measured with digital thermometers that show temperature to the nearest degree. We often see the displays from digital thermometers on banks, along with the time. The most common unit of measurement in the United States is the Fahrenheit degree ($^{\circ}\text{F}$). It comes from a measuring scale on which the freezing point of water is 32°F and the boiling point of water is at 212°F .

Sometimes the difference between the highest and lowest temperature during the day is a number of interest. This is called the **temperature range** for the day. For example, if the high for the day was 75°F and the low was 48°F , the temperature range would be 27°F .

1. If the high for a day was 94°F and the low was 59°F , what was the temperature range for the day?
2. If the high for a day was 26°F and the low was -5°F , what was the temperature range for the day?
3. If the high for a day was 21°F and the low was -10°F , what was the temperature range for the day?
4. If the high for a day was -12°F and the low was -34°F , what was the temperature range for the day?

Name _____ Date _____

LESSON ONE: Exercise 5

Football is one of the most popular games in the United States. It involves two teams that play against each other on a field that is 300 feet long. The teams alternate between **offense** and **defense**. The team on offense tries to move an oblong ball, called a football, down the field toward a goal line at the end. The team on defense tries to prevent this from happening.

While on offense, a team gets four plays to move the ball at least 10 yards down the field. Each play ends when a player with the football is tackled and goes down on the ground or when a player kicks the football toward the goal line. If the team can move the ball at least 10 yards in four or fewer plays, they receive a **first down** and are allowed four more plays. If they cannot do this, the other team takes over on offense at the end of the four plays. The success or failure of each play is measured in the number of yards that a team gains or loses. A gain in yardage is recorded as a positive number, and a loss of yardage is recorded as a negative number.

Suppose that a team gains six yards on the first play, loses four yards on the second play, gains two yards on the third play, and gains five more yards on the last play. Do they get a first down and four more plays?

Answer: $6 - 4 + 2 + 5 = 9$ No! The other team takes over on offense.

1. Suppose a team gains five yards on the first play, gains four yards on the second play, loses six yards on the third play, and gains five yards on the fourth play. Do they get a first down?
2. Suppose a team gains seven yards on the first play, gains two yards on the second play, loses two yards on the third play, and gains three yards on the fourth play. Do they get a first down?
3. Suppose a team loses four yards on the first play, gains five yards on the second play, loses two yards on the third play, and gains seven yards on the fourth play. Do they get a first down?

Name _____ Date _____

4. Suppose a team gains one yard on the first play, loses three yards on the second play, loses five yards on the third play, and gains six yards on the fourth play. Do they get a first down?

Sometimes a team will get a first down in less than four plays. In which of the following two- and three-play sets does the offense get a first down?

5. Suppose a team gains seven yards on the first play, loses four yards on the second play, and gains five yards on the third play. Do they get a first down?
6. Suppose a team gains six yards on the first play and gains seven yards on the second play. Do they get a first down?
7. Suppose a team loses three yards on the first play, loses two yards on the second play, and gains 15 yards on the third play. Do they get a first down?
8. Suppose a team loses two yards on the first play, loses four yards on the second play, and gains 17 yards on the third play. Do they get a first down?
9. Suppose a team loses four yards on the first play and gains seven yards on the second play. Do they get a first down?

Name _____ Date _____

LESSON ONE: Exercise 6

Golf is a popular game played by children, teenagers, and adults of all ages. It involves hitting a ball with a club, usually several times, until the ball goes into a hole. Each hit on the ball is called a **stroke**. Usually, there are either nine or 18 holes on a golf course.

Each hole has a number, called **par**, that is the number of strokes that a reasonably good golfer should use to get the ball into the hole. A golfer who hits the ball this many times is said to have scored a **par** on the hole. A golfer who uses one less stroke than the par is said to have scored a **birdie** on the hole, and one who uses two less strokes is said to have scored an **eagle**. If a golfer uses one more stroke than par to get the ball into the hole, he or she is said to have scored a **bogie**, and if he or she uses two strokes more than par, he or she has scored a **double-bogie**.

One way of keeping score in golf is simply to record the number of strokes that a golfer uses on each hole, and then to add them up at the end of the nine or 18 holes that are played. If several golfers are playing, the one with the smallest number of total strokes wins.

Another method of scoring is to record whether a player received a par, a birdie, an eagle, a bogie, or a double-bogie on a hole and to assign each one of these a number as follows:

eagle = -2 birdie = -1 par = 0 bogie = 1 double-bogie = 2

The final score is again obtained by adding up the scores for each hole, only now, the score can be negative, zero, or positive. If the final sum is zero, the golfer is said to have scored a **par** for that number of holes. If the sum is **negative**, the golfer is said to have scored that many strokes **under par**, and if the sum is **positive**, he or she is said to have scored that many strokes **over par**. The golfer with the most negative under par score wins, or if no one is under par, the golfer with par or the lowest over par score wins.

The following example shows the scores for each hole on a nine-hole course for two golfers, Charlie and Rosa.

Hole	Par for Hole	Charlie	Rosa
1	4	Par 0	Par 0
2	5	Par 0	Eagle -2
3	3	Par 0	Birdie -1
4	4	Bogie 1	Par 0
5	5	Double-Bogie 2	Bogie 1
6	4	Birdie -1	Par 0
7	4	Birdie -1	Par 0
8	3	Birdie -1	Bogie 1
9	5	Bogie 1	Birdie -1

Name _____ Date _____

Charlie's and Rosa's scores with respect to par are as follows:

Charlie $0 + 0 + 0 + 1 + 2 + -1 + -1 + -1 + 1 = 1$
 Charlie was one over par.

Rosa $0 + -2 + -1 + 0 + 1 + 0 + 0 + 1 + -1 = -2$
 Rosa was two under par. She beat Charlie by three strokes.

Charlie's and Rosa's scores as total strokes are as follows:

Charlie $4 + 5 + 3 + (4 + 1) + (5 + 2) + (4 - 1) + (4 - 1) + (3 - 1) + (5 + 1) = 38$

Rosa $4 + (5 - 2) + (3 - 1) + 4 + (5 + 1) + 4 + 4 + (3 + 1) + (5 - 1) = 35$

Par for course $= 4 + 5 + 3 + 4 + 5 + 4 + 4 + 3 + 5 = 37$

1. The following example shows the scores for each hole on a nine-hole course for two golfers, Leroy and Angie.

Hole	Par for Hole	Leroy	Angie
1	3	Bogie	Bogie
2	5	Par	Eagle
3	3	Par	Birdie
4	5	Bogie	Eagle
5	4	Double-Bogie	Bogie
6	4	Birdie	Par
7	4	Double-Bogie	Par
8	3	Birdie	Double-Bogie
9	5	Par	Birdie

- a) What was Leroy's score in number of strokes above or below par? _____
- b) What was Angie's score in number of strokes above or below par? _____
- c) Who won? _____
- d) What was par for the course? _____

Name _____ Date _____

e) What was Leroy's score in total number of strokes? _____

f) What was Angie's score in total number of strokes? _____

2. The following example shows the scores for each hole on a nine-hole course for two golfers, Lupi and Jose.

Hole	Par for Hole	Lupi	Jose
1	4	Par	Bogie
2	4	Par	Par
3	5	Bogie	Par
4	4	Bogie	Birdie
5	5	Bogie	Par
6	4	Birdie	Birdie
7	3	Bogie	Par
8	4	Birdie	Par
9	5	Par	Birdie

a) What was Lupi's score in number of strokes above or below par? _____

b) What was Jose's score in number of strokes above or below par? _____

c) Who won? _____

d) What was par for the course? _____

e) What was Lupi's score in total number of strokes? _____

f) What was Jose's score in total number of strokes? _____

Name _____ Date _____

LESSON ONE: Exercise 7

1. Par for each hole on some nine-hole course and the scores of four golfers are shown below.

HOLE	1	2	3	4	5	6	7	8	9
PAR	4	4	5	4	3	4	5	4	4
Sam	3	4	4	5	3	4	6	4	3
Sue	5	3	4	4	3	4	5	3	4
Mel	4	4	5	4	3	4	5	4	4
Meg	5	4	6	4	4	5	5	3	5

- a) How many strokes was par for this course?

Par = _____

- b) What were the scores in total strokes for each of the four golfers?

Sam = _____ Sue = _____ Mel = _____ Meg = _____

- c) Who won? _____

- d) What were the scores in number of strokes over or under par for each of the four golfers?

Sam = _____ Sue = _____ Mel = _____ Meg = _____

- e) Does the type of scoring affect who wins the game? _____

Name _____ Date _____

LESSON ONE: Exercise 8

An interesting type of problem involves measuring out a certain volume of liquid when you only have certain containers with which to measure volumes. As an example, consider the problem below.

Example: You have three jars with the following capacities.

Jar A = 25 quarts

Jar B = 6 quarts

Jar C = 15 quarts

You need to measure out four quarts of water into a barrel. How could you do this using only the three jars above. (You may end up with some left over water in any of the jars.)

Solution:

Fill the 25-quart jar and pour it into the barrel.

Then, pour enough water out of the barrel to fill the 15-quart jar.

Then, pour enough water out of the barrel to fill the six-quart jar.

You will be left with four quarts in the barrel.

1. You have three jars with the following capacities.

Jar A = 21 liters

Jar B = 8 liters

Jar C = 6 liters

You need to measure out 23 liters of water into a barrel. How could you do this using only the three jars above? (You may end up with some leftover water in any of the jars.)

2. You have three jars with the following capacities.

Jar A = 24 gallons

Jar B = 52 gallons

Jar C = 4 gallons

You need to measure out 20 gallons of water into a barrel. How could you do this using only the three jars above? (You may end up with some leftover water in any of the jars.)

Name _____ Date _____

LESSON TWO: NUMBER PROPERTY PROBLEMS

We often use numbers in our everyday lives without thinking about their properties. Sometimes, however, the properties of numbers can become important. They can make certain computations easier and they can tell us things that we cannot do so that we do not make foolish mistakes.

LESSON TWO: Exercise 1

Closure

Closure is an important property of number systems. It states that a number that results from a mathematical operation involving other numbers in a system will be a member of that system. We sometimes say that certain number systems are closed with respect to certain mathematical operations such as addition or multiplication.

Whole numbers and integers are closed with respect to addition and multiplication. If we add two whole numbers, the result is another whole number. If we add two integers, the result is another integer. If we multiply two whole numbers, we get another whole number and, if we multiply two integers, we get another integer. For example, 3 and 5 can both be thought of as whole numbers. The product of 3 times 5 is 15, another whole number. As another example, 4 and -6 are both integers. The sum of $4 + -6$ is -2, another integer.

1. The number of people in a room can be thought of as a whole number. If there are 10 people in a room, and five more people enter, there will be a total of how many people in the room? Is this sum another whole number?

2. The number of seats in a theater can also be thought of as a whole number. If there are 10 rows of seats in the theater, and there are eight seats in each row, how many total seats are there? Is this another whole number?

3. The number of bottles of Spring Water in a crate is a whole number.
 - a) If there are six bottles in a six-pack and 12 six-packs in a crate, how many bottles are in a crate?

Name _____ Date _____

- b) Suppose there are six bottles in a six-pack, four six-packs in a small box, and 20 small boxes in a special shipping crate. How many bottles are in a special shipping crate?
- c) Suppose there are six bottles in a six-pack, two six-packs in a gift box, six gift-boxes in a crate, and 100 crates in a delivery truck. How many bottles are there in a delivery truck?
4. Fahrenheit temperatures as measured on a digital thermometer (like the ones you see on bank clock signs) can be thought of as integer numbers. They can be positive (above zero) or negative (below zero).
- a) At 10:00 A.M. one day, the temperature was 18°F . By 3:00 P.M., the temperature had increased by 10° . What was the temperature at 3:00 P.M.? Is this temperature another integer?
- b) At 8:00 A.M. on a cold winter day, the temperature was -25°F . By noon that day, the temperature had increased by 10° . What was the temperature at noon? Is this temperature another integer?
- c) At 11:00 A.M. on a winter day, the temperature was -5°F . By 2:00 P.M. that day, the temperature had increased 10° . What was the temperature at 2:00 P.M.? Is this temperature another integer?

Name _____ Date _____

LESSON TWO: Exercise 2

Some mathematical operations are not closed in some number systems. The whole numbers are not closed with respect to subtraction. If you subtract a whole number from another whole number, you will not necessarily get a result that is another whole number. For example, $8 - 9$ is -1 , but -1 is not a whole number because it is less than zero.

1. The number of people in a room can be thought of as a whole number.
 - a) Suppose a room is initially empty. Then, 12 people enter the room. A short time later, eight people leave the room. How many people are left in the room? Is this a whole number?
 - b) Now, suppose again that a room is initially empty. Nine people enter the room, and a short time later, 11 people leave the room. Is this possible? Compute 9 minus 11 . Is the result a whole number?
2. The number of bottles of soda (pop) in a vending machine can be thought of as a whole number.
 - a) Suppose the machine is initially empty. Then, a soda distributor puts 100 bottles in the vending machine. A volleyball team comes by and purchases 25 bottles. A basketball team comes by and purchases 50 more bottles. A thirsty Cub Scout purchases six more bottles. How many bottles are left in the machine? Is this a whole number?
 - b) Suppose the machine is initially empty and then the soda distributor puts 50 bottles in it. A Girl Scout troop purchases 30 bottles, a marching band purchases 18 bottles, and a 4-H group purchases eight more bottles. Is this possible? Is $50 - 30 - 18 - 8$ a whole number?

Name _____ Date _____

LESSON TWO: Exercise 3

Unlike whole numbers, integers are closed with respect to subtraction. One integer minus another integer will always yield another integer. For example, $9 - 14 = -5$. Negative five is an integer.

1. The bank clock digital thermometer, which was mentioned on page 15, provides a good example for integer subtraction.

a) At 3:00 P.M., a digital thermometer reads 65°F . By 8:00 P.M. that evening, the temperature has decreased by 17° . What is the temperature at 8:00 P.M.? Is this temperature an integer?

b) At 4:00 P.M., a digital thermometer reads 15°F . By 10:00 P.M. that evening, the temperature has decreased by 21° . What is the temperature at 10:00 P.M.? Is this temperature an integer?

c) At noon, a digital thermometer reads -5°F . By midnight of that day, the temperature has decreased by 12° . What is the temperature at midnight? Is this temperature an integer?

2. The elevation of land surfaces is measured in terms of number of feet above sea level (a positive integer) or number of feet below sea level (a negative integer). Denver, Colorado, is sometimes called the Mile-High City because its elevation is about 5,280 feet (one mile) above sea level. Mount Elbert, the highest peak in Colorado, is 14,433 feet above sea level at its summit. Some places, such as Death Valley in California, are below sea level, so their elevations are expressed as negative integers. The lowest point in Death Valley is -282 feet below sea level. Interestingly, Mt. Whitney, the tallest mountain in the "lower 48" United States, is located just to the west of Death Valley. Its elevation is 14,495 feet above sea level.

Name _____ Date _____

- a) How much higher is Mt. Whitney than Mt. Elbert?
- b) How much higher is Mt. Elbert than Denver?
- c) How much higher is Mt. Whitney than Denver?
- d) How much higher is Mt. Whitney than the lowest point in Death Valley?
- e) How much higher is Mt. Elbert than the lowest point in Death Valley?
- f) A hiker starts at the lowest point in Death Valley and climbs up to an elevation of 2,235 feet in the foothills of Mt. Whitney. What was his gain in elevation?
- g) A mountain climber rappels down a steep cliff near Mt. Elbert in Colorado. (Rappeling is a method of descending steep cliffs, using a rope.) The top of the cliff is at an elevation of 10,100 feet above sea level. She ends her rappel on a ledge that is 9,998 feet above sea level. What was her change in elevation?
- h) Can the results of all of your calculations in this problem be thought of as integers?

Name _____ Date _____

LESSON TWO: Exercise 4

In its most general sense, division is not closed with respect to whole numbers or integers. Two special kinds of division, integer division (or whole number division) and modulo division, however, are closed with respect to both whole numbers and integers.

Integer division is a type of division in which an integer is divided by another integer, but the division is not carried out to where it generates fractional parts. Instead, the division is carried out to where it gives an integer quotient. Any remainder that is left is disregarded. Computers often perform this type of division on integer numbers.

A similar type of division, whole number division, can be used with whole numbers. Computers sometimes perform this type of division with whole numbers, which are called unsigned numbers.

Examples:

$$17 / 5 = 3$$

$$-17 / 5 = -3$$

$$23 / 6 = 3$$

Modulo division keeps what integer and whole number division disregard and disregards what they keep. In modulo division an integer is divided by another integer (or a whole number is divided by another whole number), the remainder is kept, and the quotient is disregarded. The symbol for modulo division is usually the percent sign, %.

Examples:

$$17 \% 5 = 2$$

$$-17 \% 5 = -2$$

$$23 \% 6 = 5$$

1. Whole number division can be used to determine how many full containers could be crated from a collection of items that are to be packed with a certain number of items per container. For example, how many complete six-packs of soda (pop) could be made from 51 bottles of soda?

$$51 / 6 = 8 \text{ six-packs}$$

How many bottles would be left over?

$$51 \% 6 = 3 \text{ bottles}$$

- a) Chocolate Easter rabbits are sold four to a box. How many boxes could be filled from 103 chocolate rabbits? How many rabbits would be left over?

Name _____ Date _____

- b) Eggs are sold one dozen per carton. How many cartons could be filled from 210 eggs? How many eggs would be left over?
- c) Some Christmas tree ornaments are sold 24 ornaments per box. How many boxes could be filled from 500 ornaments? How many ornaments would be left over?
2. Mary Ann has a bag of 39 quarters. If she takes them to a bank, how many dollar bills could she get? How many quarters would be left over?
3. Theophilus has a bag of 99 quarters. If he takes them to a bank, how many five-dollar bills could he get? How many quarters would be left over? How many dollar bills could he get from the left over quarters?
4. A large group of students are to be assigned to school buses for a field trip. The school buses are numbered 0, 1, 2, 3, and 4. The teacher in charge of the field trip notices that each student has a student number (assigned by the principal's office for recordkeeping) and decides to assign students to buses by modulo, dividing their student number by 5. The remainder from this division would be 0, 1, 2, 3, or 4. The students would be assigned to buses based on the remainder.

For example, a student with student number 356 would be assigned as follows:

$$356 \div 5 = 71 \text{ R } 1 \quad \text{Bus number 1}$$

The following is a list of some of the students who are going on the field trip and their student numbers. Find their bus assignments (0, 1, 2, 3, or 4).

- | | | |
|--------------|--------------|-------------|
| a) Tom 279 | b) Meg 302 | c) Jack 294 |
| d) Jose 266 | e) Lucia 255 | f) Mike 281 |
| g) Sally 272 | h) Lupi 300 | i) Jim 288 |

Name _____ Date _____

LESSON TWO: Exercise 5**Commutative Property**

Another interesting property is the commutative property. The **commutative property** of addition states that you may add together two whole numbers or integers in either order and still get the same result. For example, $2 + 3 = 5$ and $3 + 2 = 5$. This property can be stated in symbols as follows:

$$x + y = y + x.$$

We sometimes say that addition of whole numbers is commutative and that addition of integers is commutative.

1. A school bus is initially empty. Then five boys and seven girls get onto the bus. How many people are on the bus? Show that you can add the number of boys to the number of girls or add the number of girls to the number of boys and get the same answer.
2. Joan and Ann are sitting on a ledge of a vertical cliff next to the ocean. The ledge is 40 feet above the level of the sea and 50 feet below the top of the cliff. Joan climbs 30 feet up the cliff and then climbs 20 more feet up. Ann climbs 20 feet up the cliff and then climbs 30 more feet up. Will Joan and Ann end up the same number of feet above the level of the sea?
3. Jason is adding water to a small fish tank. He first pours in the contents of a three-gallon bucket, and then pours in the contents of a two-gallon water jug. Would he have poured the same amount of water into the tank if he had poured the two-gallon jug in first and then poured in the three-gallon bucket?

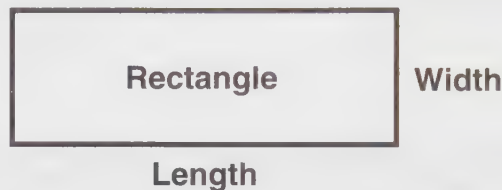
The commutative property also applies to multiplication of whole numbers and integers. For example, 5×3 gives the same product as 3×5 . This property can be stated in symbols as follows:

$$x * y = y * x.$$

The multiplication of whole numbers and the multiplication of integers is commutative.

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4. Marva, Cleostra, and Olene are counting the number of chairs in an auditorium. Marva takes a long time and counts each chair. She gets a total of 600 chairs. Cleostra notices that there are 20 rows of seats with 30 chairs in each row. She multiplies the number of rows times the number of chairs per row and gets her answer. Olene also notes that there are 20 rows with 30 chairs in each row. She multiplies the number of chairs per row times the number of rows to get her answer. Will Cleostra and Olene get the same answer as Marva?
5. The area of a rectangle (like the one shown below) can be measured in square inches. A square inch can be thought of as a small square that is one inch on each side. The number of square inches of area in a rectangle is the number of these one-inch squares that can be placed inside the rectangle.



The area in square inches can be found by multiplying the length of the rectangle (in inches) times its width (in inches). Can the area also be found by multiplying the width times the length? Show that it can be found both ways with the following rectangles.

- a) Length = 10 Width = 5
- b) Length = 8 Width = 6
- c) Length = 125 Width = 75

Unlike addition and multiplication, subtraction and division are not commutative with respect to whole numbers or integers. For example, $6 - 4$ is 2, but $4 - 6$ is -2.

6. Think of three examples that show that subtraction is not commutative for integers.
7. Think of three examples that show that division is not commutative for integers.

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LESSON TWO: Exercise 6**Associative Property**

The **associative property** refers to the order in which arithmetic operations are performed when two or more operations are present in the same calculation. It says that you will get the same answer if you perform the operations in any order.

Whole numbers and integers are associative with respect to addition. For example, $3 + 4 + 5 = 12$. You can get the answer by first adding 3 and 4 (to get 7) and then adding the 5 to the 7 (to get 12.) We show this by putting parentheses around the $3 + 4$.

$$(3 + 4) + 5 = 12$$

You could get the same answer by adding together the 4 and the 5 (to get 9) and then adding the 3 to the 9 (to get 12.) We show this by placing parentheses around the $4 + 5$.

$$3 + (4 + 5) = 12$$

You get the same answer, regardless of the order in which the addition operations are done.

$$(3 + 4) + 5 = 3 + (4 + 5) = (3 + 5) + 4 = 12$$

1. Anna has three fish tanks. There are five guppies in the first tank, seven goldfish in the second tank, and three mollies in the third tank. How many fish does Anna have in all three of the tanks? Show that you will get the same answer if you **(1)** add 5 plus 7 and then add 3, or **(2)** add 7 plus 3 and then add 5, or **(3)** add 5 plus 3 and then add 7.
2. There are three teams of students from Pleasant Hill Junior High School competing at a Science Fair. Four students have prepared a project on the growth of plants in different types of soils, two more students have written a computer program that finds prime numbers in a neat way, and three more students have built a rocket. How many students from Pleasant Hill Junior High School are competing in the science fair? Show that you will get the same answer if you **(1)** add 4 plus 2 and then add 3, or **(2)** add 2 plus 3 and then add 4, or **(3)** add 4 plus 3 and then add 2.

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3. Clarence is counting the number of tools in his father's workshop. He finds five screwdrivers, two hammers, three saws, and 10 wrenches. How many tools are in Clarence's father's workshop? Show that you will get the same answer if you **(1)** add 5 plus 2 to get 7, then add 3 plus 10 to get 13, and then add the 7 and 13 to get 20, or **(2)** add 5 plus 3 to get 8, then add 2 plus 10 to get 12, then add 8 plus 12 to get 20. Can you think of other ways to add these numbers, two at a time, that will give you the correct answer?

Multiplication is also associative with respect to whole numbers and integers. For example, $3 \times 4 \times 5$ can be computed by first multiplying 3 times 4 to get 12 and then multiplying 12 by 5 to get the final answer, 60. It can also be done by multiplying 4 times 5 to get 20, and then multiplying the 20 times 3 to get 60. The following expression shows that these two calculations give the same answer. (The multiplications in parentheses are done first.)

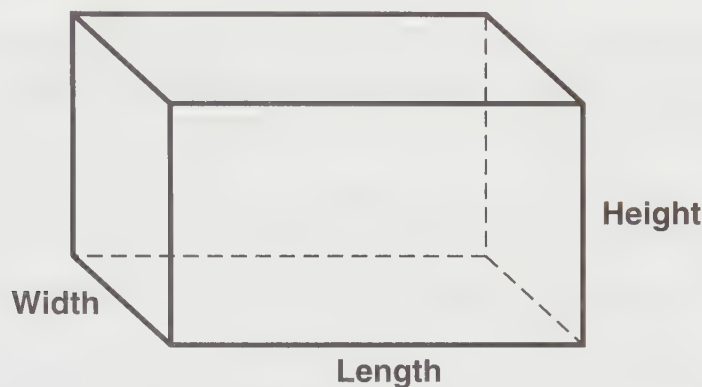
$$(3 \times 4) \times 5 = 3 \times (4 \times 5) = 60$$

4. Elmer needs to find the total number of marbles in a crate. The marbles are stored in bags, and the bags are stored in small boxes. The small boxes are packed in the crate. There are 12 marbles per bag. Five bags are stored in each small box. Twenty-four small boxes are packed into each crate. Find three ways to multiply to find the total number of marbles in the crate. Do you get the same answer each way?
5. Ellen was trying to find the number of people eligible to run in an election for a special class leader position at her school. To be eligible, a person had to be an officer in a school organization. There were three grades in her school: seventh, eighth, and ninth. Each class had five organizations, and each organization had four officers: President, Vice President, Secretary, and Treasurer. Find three ways to multiply to find the total number of students eligible to run in the election. Do you get the same answer each way?

Name _____ Date _____

LESSON TWO: Exercise 7

1. The volume of a rectangular box (like the one shown below) can be measured in cubic inches. A cubic inch can be thought of as a small cube that is one inch on each side. The number of cubic inches of volume in a rectangular box is the number of these one-inch cubes that can be stacked inside the rectangular box. The volume in cubic inches can be found by multiplying the length of the box (in inches) times its width (in inches) times its height (in inches).



- a) Show three ways to find the volume of a rectangular box with the following dimensions. Do they all three yield the same answer?

Length = 12 inches

Width = 8 inches

Height = 6 inches

- b) Show three ways to find the volume of a rectangular box with the following dimensions. Do they all three yield the same answer?

Length = 110 inches

Width = 75 inches

Height = 20 inches

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LESSON TWO: Exercise 8**Distribution**

Another property of numbers is the **distributive property**. It becomes important when two operations, such as addition and multiplication, are both present in a problem. The following example shows such a problem.

$$2 \times (3 + 5) = 16$$

In this problem, the parentheses are used to indicate which operation is to be done first. Therefore, you would first add 3 plus 5 to get 8, and then multiply 8 by 2 to get the final answer, 16.

The distributive property says that a problem, like the one above, can be written as the sum of two multiplication operations:

$$2 \times (3 + 5) = (2 \times 3) + (2 \times 5) = 16$$

To evaluate the right-hand form of the problem, you would multiply 2 times 3 to get 6, then multiply 2 times 5 to get 10, and then add the products 6 and 10 to get the final answer, 16.

We say that **multiplication distributes over addition** for integers and whole numbers.

1. Marjorie and Janet are working in a carnival at a county fair. A person who buys a special money-saver ticket at the carnival gate gets to play seven coin-toss games and five dart-throw games. A man purchases six special money-saver tickets. How many games will he be able to play? Marjorie thinks that the best way to find the answer is to add 7 plus 5 and then to multiply the sum times 6. Janet feels that the best way to get the answer is to multiply 7 times 6, then to multiply 5 times 6, and then add the two products together. Calculate the answer Marjorie's way and then Janet's way. Do you get the same answer? Is this an application of the distributive property?
2. Yuri works at a pet store on weekends. One weekend, the store was having a special. Every customer who purchased two gerbils got five goldfish for free. Fifty-two customers took advantage of the special that weekend. How many total animals (gerbils and goldfish) did the customers get? Think of two ways to compute the value. Do you get the same answer both ways?

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3. Trang was waiting for a relative to arrive at the airport. While he was waiting, he noticed 11 commuter airplanes take off, headed for a nearby city. He overheard an airport employee say that each of the airplanes was full and that each one carried 30 passengers and four crew members. How many total people left for the nearby city on commuter airplanes while Trang was watching? Think of two ways to get the answer.
4. The distributive property can sometimes be used to make arithmetic computations easier. For example, suppose that you had to multiply 15 times 12. You could probably do this, but you can make it easier if you notice that 12 is $10 + 2$ and then rewrite the problem as shown below.

$$15 \times 12 = 15 \times (10 + 2) = 15 \times 10 + 15 \times 2$$

Fifteen times 10 is 150 and 15 times 2 is 30. The sum of 150 and 30 is the answer, 180.

- a) Calculate 20×55 using the method described above.
- b) Calculate 19×110 using the method described above.
- c) Calculate 14×202 using the method described above.

The distributive property also applies to subtraction and division in the following ways. Multiplication distributes over subtraction, division distributes over addition, and division distributes over subtraction. Some examples are shown below.

$$3 \times (7 - 4) = (3 \times 7) - (3 \times 4) = 21 - 12 = 9$$

$$(8 + 6) / 2 = (8 / 2) + (6 / 2) = 4 + 3 = 7$$

$$(18 - 12) / 3 = (18 / 3) - (12 / 3) = 6 - 4 = 2$$

5. A fish tank contains 24 black mollies (a type of fish) and 18 silver mollies. One-third of the fish of each color are males. How many male fish are in the tank? Think of two ways to compute your answer.

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6. Mandy is checking apples in a barrel. Her mother tells her to remove one-third of the good apples and put them in a bowl on the dining room table. Mandy finds that there are 100 apples in the barrel and that 28 of them are bad. How many apples will Mandy put in the bowl on the dining room table?
7. Sylvester is working for his aunt, assembling bird houses that she sells in her craft store. His aunt inspects all of the birdhouses that he assembles and classifies them as good or faulty. Sylvester gets paid \$2 for every good birdhouse. Out of 20 birdhouses that he assembles, six are found to be faulty. How much does Sylvester get paid?
8. You can use the distributive property involving multiplication and subtraction to simplify computations, like you used the one involving multiplication and addition. For example, suppose that you had to multiply 20 times 49. You might make the computation easier if you notice that 49 is 50 minus 1 and then rewrite the problem as shown below.

$$20 \times 49 = 20 \times (50 - 1) = (20 \times 50) - (20 \times 1)$$

Twenty times 50 is 1,000 and 20 times 1 is 20. The difference of 1,000 minus 20 is the answer, 980.

- a) Calculate 8×79 using the method described above.
- b) Calculate 19×99 using the method described above.
- c) Calculate 5×208 using the method described above.

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LESSON TWO: Exercise 9**Arithmetic Expressions**

Arithmetic expressions are calculations that contain at least one arithmetic operator. The more complicated ones may contain several arithmetic operators. The calculation below is a typical example.

$$3 \times 5 - 6 / 2 + 4 \times 6 - 8 \times 2$$

When you have to evaluate an expression like this, you must remember something called **operator precedence**.

The * or x (multiplication) and / (division) operations have the highest precedence and are therefore done first in arithmetic expressions. They are usually done from left to right across an expression. The + (add) and - (subtract) operations have a lower precedence and are done after the multiplication and division ones have been completed. These operations are also generally done from left to right across the expression. The expression shown above would be evaluated first by multiplying 3 times 5 (to get 15), then dividing 6 by 2 (to get 3), then multiplying 4 times 6 (to get 24), and then multiplying 8 times 2 (to get 16.) Next, the 3 would be subtracted from the 15 (to get 12), the 24 would be added to the 12 (to get 36), and the 16 would be subtracted from the 36 to get the final answer 10.

$$\begin{aligned} 3 \times 5 - 6 / 2 + 4 \times 6 - 8 \times 2 &= 15 - 6 / 2 + 4 \times 6 - 8 \times 2 \\ &= 15 - 3 + 4 \times 6 - 8 \times 2 \\ &= 15 - 3 + 24 - 8 \times 2 \\ &= 15 - 3 + 24 - 16 \\ &= 12 + 24 - 16 \\ &= 36 - 16 \\ &= 20 \end{aligned}$$

Parentheses can be used to change this order of precedence. If a portion of a calculation is enclosed in parentheses, the part within the parentheses is evaluated first. The regular order of precedence is used to evaluate the computations inside the parentheses. Then, the parts of the computation outside the parentheses are calculated. The example below shows how this is done.

$$\begin{aligned} 3 * (8 / 2 + 3 * 5) - 3 * 6 + 4 &= 3 * (4 + 15) - 3 * 6 + 4 \\ &= 3 * 19 - 3 * 6 + 4 \\ &= 57 - 3 * 6 + 4 \\ &= 57 - 18 + 4 \\ &= 39 + 4 \\ &= 43 \end{aligned}$$

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If more than one level of parentheses is present, the calculations inside the innermost level are done first, and so on. You work from the innermost level, outward to the outermost level.

1. Evaluate the following arithmetic expressions.

a) $15 / 3 + 14 / 7 + 6 / 2$

b) $2 * 6 - 3 * 9 + 4 * 4$

c) $12 * 3 - 128 / 4 - 3 * 7 + 6$

d) $6 * 7 * 5 - 3 * 4 * 6$

e) $6 / 3 / 2$

f) $6 / 2 * 3$

g) $90 / 18 * 5 + 4 * 3 / 6 - 18 / 6 + 1$

h) $5 * (7 - 4) + 3$

i) $6 * (14 - 2 * 3) - 5$

j) $(6 - 4) * (3 * 5 - 11)$

k) $2 * (7 * 2 - 5) / (6 - 4) + 3 * 8$

l) $5 * (2 + (3 + 4))$

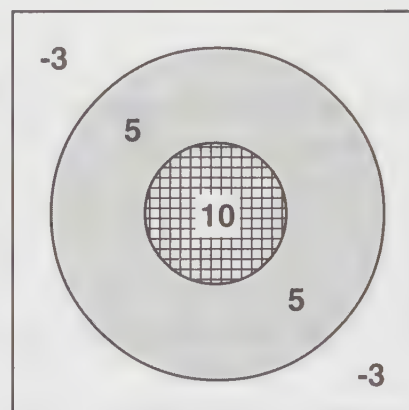
m) $(3 + 2 * (8 - 5) + 1) / 2$

n) $(11 - 3 * (21 - 4 * (2 + 3) + 1)) * 2$

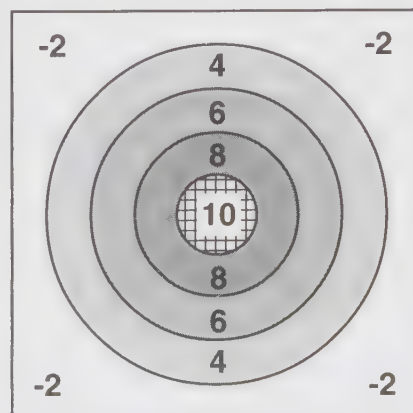
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2. Lurlene is checking Christmas light bulbs before stringing the lights on the Christmas tree. Her mother tells her that she can use one-third of the red light bulbs, one-half of the green light bulbs, and one-fourth of the blue light bulbs. Lurlene has a box of 27 red bulbs, a box of 24 green bulbs, and another box of 32 blue bulbs. How many total bulbs can she use? Write a single arithmetic expression that, when evaluated, will give the right answer.
3. Puppyland, a dog kennel and dog store, has a collection of Labrador Retriever (Lab) puppies. They come in three colors: black, chocolate, and golden. In one kennel run are 8 black Labs, 12 chocolate Labs, and six golden Labs. In each color, there are equal numbers of male and female puppies. How many (total) female Lab puppies are there? Think of two ways to compute your answer.

4. Buck is target shooting with his BB gun. He has a target that assigns 10 points to each shot that hits the bull's-eye and five points to each shot that hits the target ring that surrounds the bull's-eye. Minus three (-3) points are given for each shot that is outside the ring that surrounds the bull's-eye. Buck puts 31 shots in the bull's-eye, 17 shots in the ring around the bull's-eye, and 9 shots outside the ring. How many points did Buck get?



5. Buck shoots again the next day at a fancier target. This target assigns 10 points for each bull's-eye, eight points for each shot in the first ring from the bull's-eye, six points for the second ring from the bull's-eye, and four points for each shot in the third ring from the bull's-eye. A minus two (-2) points are given for each shot outside the third ring. Buck shoots 75 shots. He puts 29 shots in the bull's-eye, 21 shots in the first ring, 10 shots in the second ring, and seven shots in the third ring. How many shots were outside the third ring? What was Buck's score?



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LESSON THREE: EXPONENTIAL PROBLEMS**LESSON THREE: Exercise 1****Exponents**

In many mathematical problems we see things like 3^2 or 2^4 . These are arithmetic terms that contain exponents. **Exponents** are powers to which other numbers, called **bases**, are raised. In 3^2 , the **3** is the base and the **2** is the exponent. In 2^4 , the **2** is the base and the **4** is the exponent.

To evaluate a number with an exponent, you multiply the base times itself a number of times equal to the exponent. For example, 3^2 would be evaluated as follows:

$$3^2 = 3 * 3 = 9$$

The term 2^4 would be computed as shown below:

$$2^4 = 2 * 2 * 2 * 2 = 16$$

We have special names for some exponents. A number raised to the **2** power is called **the square** of the number and a number raised to the **3** power is called the **cube** of the number.

$$5^2 = 25$$

$$10^2 = 100$$

$$6^3 = 216$$

$$10^3 = 1000$$

A number to the **1** power is just that number.

$$5^1 = 5$$

$$12^1 = 12$$

$$200^1 = 200$$

A special case of numbers with exponents occurs when the exponent is equal to 0. *Any nonzero number to the power 0 is 1.*

$$1^0 = 1$$

$$2^0 = 1$$

$$8^0 = 1$$

$$100^0 = 1$$

1. Evaluate the following exponential numbers.

a) 3^3

b) 4^2

c) 2^5

d) 5^3

e) 3^5

f) 6^4

g) 15^0

h) 9^1

i) 5^0

j) 13^1

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2. Find the squares of the following numbers.

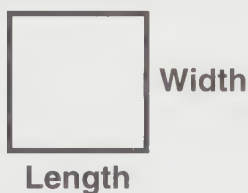
- a) 2 b) 3 c) 4 d) 7 e) 9 f) 11

3. Find the cubes of the following numbers.

- a) 2 b) 3 c) 4 d) 5 e) 8 f) 12

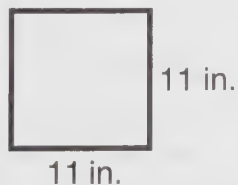
A square, shown below, is a geometrical figure whose length is the same as its width. If the length and width of a square are measured in inches, its area can be expressed in the square inch units described in Lesson Two. The area of a square is just the square of its length or width dimension.

length = width



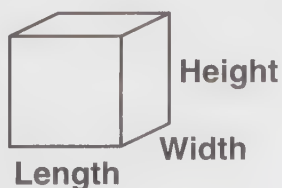
4. Find the areas of the following squares.

- a) A square with width = 5 in. b) A square with length = 9 in.
- c) A square with width = 14 in. d) A square with length = 115 in.
- e) The square in the figure below.



A cube is also a geometrical figure. Its length, its width, and its height are all equal. The volume of a cube is just the square of its length, width, or height dimension.

length = width = height



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5. Find the areas of the following cubes.

a) A cube with width = 2 inches

b) A cube with length = 5 inches

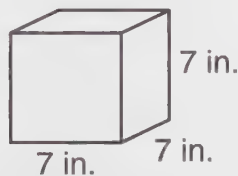
c) A cube with height = 8 inches

d) A cube with length = 15 inches

e) A cube with width = 105 inches

f) A cube with height = 200 inches

g) A cube in the figure below.



6. In a classroom spelling contest, the teacher divides the class into five teams. Each team has five members. The teacher decides to ask each student on each team five spelling words. How many spelling words will he need to ask?

7. Some Girls Scouts were collecting cans of food for needy people at Christmas. Twelve troops of Girl Scouts participated in the collection. Each troop had 12 members and was required to collect enough cans to fill 12 boxes. Each box held 12 cans. How many cans of food were collected?

Name _____ Date _____

LESSON THREE: Exercise 2

An important type of problem that can be solved with exponentials is called a **doubling problem**. A doubling problem is a problem in which the size, number, or amount of something doubles at the end of some time period. At the end of several time periods it will have doubled several times. The following example with bacteria shows how such a problem works.

Bacteria are tiny creatures that reproduce by splitting themselves in half. They are so small that, to see them, we have to look at them through a microscope. Suppose that there is a colony of bacteria that doubles in number every day. If the colony starts with one bacterium (bacterium is the singular form of bacteria) on Day 0, there will be $2 * 1 = 2$ bacteria at the end of Day 1, $2 * 2 * 1 = 4$ bacteria at the end of the second day (Day 2), $2 * 2 * 2 * 1 = 8$ bacteria at the end of the third day, and so on. We can write the number of bacteria present on any day as an exponential term with the base equal to two and the exponent equal to the day number.

Day 0	$2^0 = 1$
Day 1	$2^1 = 2$
Day 2	$2^2 = 4$
Day 3	$2^3 = 8$
Day 4	$2^4 = 16$
Day 5	$2^5 = 32$
.	
.	
.	

This type of behavior is sometimes called **exponential growth**.

1. In the bacteria problem above:

- How many bacteria will be present at the end of six days? _____
- How many bacteria will be present at the end of seven days? _____
- How many bacteria will be present at the end of eight days? _____

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2. A type of flu has hit River City Junior High School. Each day, the number of students with the flu disease doubles. On Monday, only one student has the flu.

- a) How many students will have the flu on Tuesday? _____
- b) How many students will have the flu on Wednesday? _____
- c) How many students will have the flu on Thursday? _____
- d) How many students will have the flu on Friday? _____

3. Marcus' older brother has started a computer business. The business grows so fast that the number of customers that his brother serves doubles each month. His brother had only one customer in January.

- a) How many customers did he have in February? _____
- b) How many customers did he have in March? _____
- c) How many customers did he have in April? _____
- d) How many customers did he have in May? _____
- e) How many customers did he have in June? _____
- f) How many customers did he have in August? _____
- g) How many customers did he have in October? _____
- h) How many customers did he have in December? _____

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4. A monster in a movie is called The Glob. Its size is measured as its volume in cubic inches. It is a ferocious monster that eats everything in its path. Because of its big appetite, its size doubles every hour. Suppose its size is initially one cubic inch.

- a) How big is it after one hour? _____
- b) How big is it after two hours? _____
- c) How big is it after four hours? _____
- d) How big is it after 12 hours? _____
- e) How big is it after 24 hours? _____

5. Daniel's grandfather wants to invest some money so that Daniel will have money to go to college when he graduates from high school. His grandfather finds a really great investment that doubles the amount of money invested at the end of each year. When Daniel is born (zero years old), his grandfather invests one dollar.

- a) How many dollars will be in the investment when Daniel is one year old? _____
- b) How many dollars will be in the investment when Daniel is two years old? _____
- c) How many dollars will be in the investment when Daniel is three years old? _____
- d) How many dollars will be in the investment when Daniel is five years old? _____
- e) How many dollars will be in the investment when Daniel is 10 years old? _____
- f) How many dollars will be in the investment when Daniel is 18 years old and graduates from high school? _____

Name _____ Date _____

LESSON THREE: Exercise 3

The doubling problem also applies to problems that do not start with an amount, a size, or a number equal to one. For example, suppose that there was a colony of bacteria whose number doubled each day. In this example, the colony starts with 100 bacteria on Day 0. There would be 200 bacteria on Day 1, 400 bacteria on Day 2, and so on. The numbers of bacteria on the different days could be found in the following way.

Day 0	$100 \times 2^0 = 100$
Day 1	$100 \times 2^1 = 200$
Day 2	$100 \times 2^2 = 400$
Day 3	$100 \times 2^3 = 800$
Day 4	$100 \times 2^4 = 1,600$

.
.
.

1. In the bacteria problem above:

- a) How many bacteria will be present at the end of five days? _____
- b) How many bacteria will be present at the end of six days? _____
- c) How many bacteria will be present at the end of eight days? _____
- d) How many bacteria will be present at the end of 10 days? _____

2. A type of flu has hit River City Junior High School. Each day, the number of students with the flu disease doubles. On Monday, 25 students have the flu.

- a) How many students will have the flu on Tuesday? _____
- b) How many students will have the flu on Wednesday? _____
- c) How many students will have the flu on Thursday? _____
- d) How many students will have the flu on Friday? _____

Name _____ Date _____

3. Marcus' older brother has started a computer business. The business grows so fast that the number of customers that his brother serves doubles each month. His brother had 5 customers in January.

- a) How many customers did he have in February? _____
- b) How many customers did he have in March? _____
- c) How many customers did he have in April? _____
- d) How many customers did he have in May? _____
- e) How many customers did he have in June? _____
- f) How many customers did he have in August? _____
- g) How many customers did he have in October? _____
- h) How many customers did he have in December? _____

4. A monster in a movie is called The Glob. Its size is measured as its volume in cubic inches. It is a ferocious monster that eats everything in its path. Because of its big appetite, its size doubles every hour. Suppose its size is initially 500 cubic inches.

- a) How big is it after one hour? _____
- b) How big is it after two hours? _____
- c) How big is it after four hours? _____
- d) How big is it after 12 hours? _____
- e) How big is it after 24 hours? _____

Name _____ Date _____

5. Daniel's grandfather wants to invest some money so that Daniel will have money to go to college when he graduates from high school. His grandfather finds a really great investment that doubles the amount of money invested at the end of each year. When Daniel is born (zero years old), his grandfather invests 100 dollars.

- a) How many dollars will be in the investment when Daniel is one year old? _____
- b) How many dollars will be in the investment when Daniel is two years old? _____
- c) How many dollars will be in the investment when Daniel is three years old? _____
- d) How many dollars will be in the investment when Daniel is five years old? _____
- e) How many dollars will be in the investment when Daniel is 10 years old? _____
- f) How many dollars will be in the investment when Daniel is 18 years old and graduates from high school? _____

6. John's grandfather also wants to invest some money so that John will have money to attend college when he graduates from high school. He finds an investment in which the amount of money invested doubles at the end of every three years. When John is born (zero years old), he invests 100 dollars.

- a) How many dollars will be in the investment when John is three years old? _____
- b) How many dollars will be in the investment when John is six years old? _____
- c) How many dollars will be in the investment when John is nine years old? _____
- d) How many dollars will be in the investment when John is 12 years old? _____
- e) How many dollars will be in the investment when John is 15 years old? _____
- f) How many dollars will be in the investment when John is 18 years old and graduates from high school? _____

LESSON THREE: Exercise 4

1. Linus has gone fishing with his father in the North Woods. The days are very nice, but each evening they are attacked by clouds of mosquitoes. Their fishing guide tells them that the number of mosquitoes in this particular area can quadruple (increase by a factor of four) each week. Suppose that there were initially 100 mosquitoes in the area (in week zero.)

- How many mosquitoes would there be at the end of week one? _____
- How many mosquitoes would there be at the end of week two? _____
- How many mosquitoes would there be at the end of week three? _____
- How many mosquitoes would there be at the end of week six? _____

a) Slick Sal initially bets \$1,000 and then plays the card game five times. Each time she leaves her previous bet plus her current winnings as the bet for the next game. How much money does she have at the end of the five games?

3. The population of a certain country triples every 20 years. The population is 4 million initially.

- a) What is the population 20 years later? _____
- b) What is the population 40 years later? _____
- c) What is the population 80 years later? _____

Name _____ Date _____

LESSON THREE: Exercise 5

Another type of problem that can be solved with exponentials is one in which a certain number of symbols is used to represent something. For example, suppose the letters A, B, C, and D, were used to represent things. If only a single letter were used, four things could be represented.

A B C D

A could stand for one thing, **B** for another thing, **C** for another thing, and **D** for yet another thing. The number of things represented would be 4 to the one power, 4^1 .

If two letters were used, the number of things represented would be much larger.

AA AB AC AD
BA BB BC BD
CA CB CC CD
DA DB DC DD

The number of things represented would now be 4^2 or **16**. In general, the number of things that can be represented is expressed by an exponential with the base equal to the number of symbols that are used (4 in the above example) and the exponent equal to the number of symbols that are used in each item (2 in the above example.)

1. A certain very small country decides to use the letters A, B, C, D, and E on its automobile license plates. It uses three columns of letters on each plate. (A sample license plate is shown below.) How many different license plates can it have?

SOMEWHERE
BCD
JAN 2000

2. A country decides to use all 26 letters of the alphabet on its automobile license plates. It uses four columns of letters on each plate. (A sample license plate is shown below.) How many different license plates can it have? (Ask your teacher to show how to calculate this number on your calculator.)

SOMEWHERE
XAMS
JAN 2000

Name _____ Date _____

3. A teacher decides to give each of her students an identification code that has four letters. Each of the four letters can be X, Y, or Z. How many different student codes can she give?

The number system that we use to count and specify amounts and sizes is called the **decimal system**. In this system, we use ten digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If only **one column** of numbers is used, only $10^1 = 10$ different numbers can be represented. They range from **0** to **9**. The smallest value that can be represented is **0** and the largest is **9**.

4. To represent numbers larger than 9, we use more than one column of digits. For example, if **two columns** are used, numbers from **00** to **99** can be represented. A total of $10^2 = 100$ numbers can be represented. The smallest number is **00** and the largest one is **100 - 1 = 99**.

a) How many numbers can be represented if three columns are used? _____

What are the largest and smallest numbers? _____

b) How many numbers can be represented if four columns are used? _____

What are the largest and smallest numbers? _____

Name _____ Date _____

LESSON THREE: Exercise 6

1. (Do problem 4 on page 43 before you attempt this problem.) It is easy to convert a number that is expressed in the octal system to a decimal number. The value of an octal number in the decimal system is obtained by multiplying each octal digit by the value of the column in which it is placed. The right-most column has a column value of $8^0 = 1$, the next column to the left has a column value of $8^1 = 8$, and the column to the left of that has a column value of $8^2 = 64$. The example below shows how you can convert an octal number to decimal. (An octal number is written with ₈ as a subscript.)

$$\begin{aligned} 436_8 &= 4 \times 8^2 + 3 \times 8^1 + 6 \times 8^0 \\ &= 4 \times 64 + 3 \times 8 + 6 \times 1 \\ &= 256 + 24 + 6 \\ &= 286 \end{aligned}$$

- a) Convert the octal number 74_8 to decimal.
- b) Convert the octal number 367_8 to decimal.
- c) Convert the octal number 15_8 to decimal.
- d) Convert the octal number 702_8 to decimal.
2. (Do problem 1 on page 44 before you attempt to do this problem.) The number system used by computers is called the **binary system**. It uses only the digits 0 and 1. If only **one column** of numbers is used, $2^1 = 2$ different numbers can be represented. They range from **0** to **1**. The smallest value that can be represented is **0** and the largest is **1**.
- To represent numbers larger than 1, more than one column of digits is used. If **two columns** are used, numbers from **00** to **11₂** can be represented. (The **11₂** does not represent 11. It is a binary value that is equivalent to the number 3 in decimal. The subscript **2** indicates that it is a binary number.) A total of $2^2 = 4$ numbers can be represented. The smallest number is **00** and the largest one is **4 - 1 = 3**.

Name _____ Date _____

a) How many numbers can be represented if three columns are used? _____

What are the largest and smallest numbers? _____

b) How many numbers can be represented if four columns are used? _____

What are the largest and smallest numbers? _____

3. (Do problem 2 on page 44 before you attempt this problem.) You can easily convert a number that is expressed in the binary system to a decimal number. The value of a binary number in decimal is obtained by multiplying each binary digit (0 or 1) by the value of the column in which it is placed. The right-most column has a column value of $2^0 = 1$, the next column to the left has a column value of $2^1 = 2$, the column to the left of that has a column value of $2^2 = 4$, and so on. The example below shows you how you can convert a binary number to decimal.

$$\begin{aligned} 101_2 &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 4 + 0 + 1 \\ &= 5 \end{aligned}$$

a) Convert the binary number 11_2 to decimal.b) Convert the binary number 110_2 to decimal.c) Convert the binary number 111_2 to decimal.d) Convert the binary number 1011_2 to decimal.

Name _____ Date _____

LESSON THREE: Exercise 7**Negative Exponents**

It is possible for the exponent of a base number to be negative. For example, the terms, 3^{-5} and 4^{-2} have negative exponents. Terms with negative exponents can be written as 1 divided by the base with a positive exponent.

$$3^{-5} = \frac{1}{3^5} \qquad 4^{-2} = \frac{1}{4^2}$$

1. Represent the following numbers with negative exponents as fractions. For example,

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

- a) 2^{-2} b) 3^{-2} c) 6^{-2}
d) 5^{-1} e) 3^{-4} f) 4^{-3}

2. A rectangle of plastic has an area of 24 square inches. If the rectangle is cut in half, each of the halves will have an area of 12 square inches ($24 \times \frac{1}{2} = 24 \times 2^{-1}$.) If each of these halves is cut in half, the pieces will have areas of 6 square inches ($24 \times \frac{1}{2} \times \frac{1}{2} = 24 \times 2^{-2}$.) Suppose that each of the pieces is cut in half again.

- a) What is the area of each of the pieces (in square inches)?

- b) Write the area from (a) as 24 times a number with a negative exponent.

3. (Work problem 2 before attempting this problem.) Trang is helping his uncle cut plywood for a home improvement job. His uncle needs some pieces of plywood that are squares, one-foot by one-foot in size. The area of each piece of these squares is one square foot. The uncle tells Trang to start with a piece of plywood that is eight feet long by four feet wide, and cut it in half. Then, he tells Trang to cut each of the halves in half. Then, he tells Trang to cut each of the pieces in half again. The uncle tells Trang to cut each of those pieces in half, then cut each of those pieces in half one more time.

- a) What was the area of the original eight- by four-foot piece of plywood?

Name _____ Date _____

- b) Are the final pieces one foot squares?
 - c) What is the area of each of the final pieces?
 - d) Write the area from (c) as the area from (a) times a number with a negative exponent.
 - e) Draw an eight-inch by four-inch rectangle on a piece of paper. This is a scale model of the eight- by four-foot piece of plywood. Then draw lines where each of the cuts in the problem would have been made (on the scale model.) Measure the size of the final pieces. Are they one- by one-inch squares?
4. Mindy likes chocolate pie. Her mother bakes a large chocolate pie for her one day and puts it in the refrigerator. Her mother tells Mindy that on any day she can have only half of however much of the pie is left. On the first day, Mindy gets half a pie.
- a) What fraction of the pie does Mindy get on the second day? Write this as a number with a negative exponent.
 - b) What fraction of the pie does she get on the third day? Write this as a number with a negative exponent.
 - c) What fraction of the pie does she get on the fourth day? Write this as a number with a negative exponent.
 - d) What fraction of the pie does she get on the fifth day? Write this as a number with a negative exponent.
 - e) After the fifth day, what fraction of the pie is left?

Name _____ Date _____

5. Vanessa reads that the rain forest in a tropical country is being destroyed at an alarming speed. Each year, one-third of the remaining forest is cut down and sold as lumber. In the year 2000 there were 24,300 acres of rain forest in the country.

a) How many acres were cut down by 2001?

How many acres were left? (Hint: The number of acres left is equal to the original number of acres minus the number of acres that were cut down.)

b) How many acres were cut down between 2001 and 2002? (Hint: Use the number of acres left from part (a) as your original number.)

How many acres were left?

c) How many acres were cut down between 2002 and 2003?

How many acres were left?

6. If you flip a coin, the probability that you will get a head is $\frac{1}{2}$ (and the probability that you will get a tail is also $\frac{1}{2}$.) This can be written as 2^{-1} , and it means that there is one chance out of 2 that you will get a head. If you flip a coin more than once, the probability that you will get a head on every flip is 2 to some negative power. The power is the number of times that the coin was flipped. For example, if you flip a coin twice, the probability that you will get two heads is $2^{-2} = \frac{1}{2} = \frac{1}{4}$. This means that there is one chance out of four that you will get two heads.

a) What is the probability that you will get three heads if you flip a coin three times?

b) What is the probability that you will get four heads if you flip a coin four times?

c) What is the probability that you will get five heads if you flip a coin five times?

Name _____ Date _____

LESSON FOUR: FACTORING PROBLEMS**LESSON FOUR: Exercise 1****Factors**

Many interesting mathematical problems that involve whole numbers can be solved by looking at the **factors** of the numbers. Numbers that are multiplied to yield a product are called factors. For example, if you multiply 2 times 7 to get 14, 2 and 7 are factors of 14. There can be many factors of a given number as the example below shows.

Find the factors of 24:

$$\begin{array}{l} 1 * 24 = 24, \quad 2 * 12 = 24, \quad 3 * 8 = 24, \quad 4 * 6 = 24 \\ 6 * 4 = 24, \quad 8 * 3 = 24, \quad 12 * 2 = 24, \quad 24 * 1 = 24 \end{array}$$

The factors of 24 are all of the numbers that are found in the above products. The factors of 24 are therefore, 1, 2, 3, 4, 6, 8, 12, and 24.

1. Marcella was trying out different ways of arranging the chairs in her eighth grade classroom. There were 36 chairs in the classroom. Marcella noticed she could place them in four rows with nine chairs in each row. In what other ways could she arrange the chairs?
2. The director of a high school marching band wanted his band to march in many different ways while they marched in a parade. There were 48 musicians in the band and he always wanted to have them marching in rows that were all the same length. He decided to have them start out marching single file, and then to move into a column of twos (i.e. two musicians in a row.) What other ways could the band be marched? Write the number of musicians per row and the number of rows that the band would have.
3. There were 40 students in a seventh grade class. The teacher decided to break the class into teams for a math contest. Each of the teams should have the same number of students. She noted that she could get two teams with 20 people on each team, but the teams would be too large. What other ways could she divide the class? Write the number of teams and the number of people per team.

Name _____ Date _____

LESSON FOUR: Exercise 2**Prime Factors**

An interesting group of whole numbers are called **prime numbers**. A whole number is called a **prime number** if it is divisible only by 1 and by itself. The number 7 is a prime number because it can only be divided by 1 and by 7. The number 6, however, is not a prime number because it can be divided by 2 and 3 in addition to 1 and 6.

The **prime factors** of a whole number are the factors (other than one and the number itself) that are prime numbers. For example, the factors of 18 include 1, 2, 3, 6, 9, and 18. The factors that are prime numbers and whose product produces the original number are prime factors. Since $2 \times 3 \times 3 = 18$, the prime factors are 2, 3, and 3.

One way to find prime factors is with a **factor tree**. Using a factor tree, you can divide a whole number into two factors, then divide factors into factors, and continue to do this until you have nothing but prime factors. The original number and each of the factors are shown as a circle with a number in it. Factors are connected to their product by a line. The example shows the factor tree for the number 24.



The numbers in the circles that do not have any lines leaving them are the prime factors, in this case, 2, 2, 2, and 3.

1. Which of the following numbers are prime numbers? (Write prime or not prime.)

a) 13

b) 21

c) 36

d) 7

e) 39

f) 45

g) 9

h) 72

i) 81

2. If two prime numbers differ by 2, they are sometimes called **twin primes**. Five and seven are twin primes and so are 11 and 13. Can you find any other twin primes less than 100?

Name _____ Date _____

3. What is the smallest nonzero (a positive integer) whole number that is evenly divisible by 3, 4, and 5?
4. What is the smallest nonzero whole number that is evenly divisible by 2, 3, 4, 5, 6, 7, 8, and 9?
5. A gym teacher is trying to divide her class into teams. She finds that if she divides the students into two groups, three groups, or four groups of equal size, she always has one student left over. What is the smallest number of students she could have in her class?
6. A bookstore is having a sale on school supplies. The owner wants to sell some loose pencils in packages. He finds that if he divides his pencils into three, four, or five packages of equal size, he always has two pencils left over. What is the smallest number of pencils that he can have?
7. Draw the factor trees for the following numbers.

a) 18

b) 36

c) 45

d) 64

e) 96

f) 121

Name _____ Date _____

LESSON FOUR: Exercise 3

1. A mathematician claims that a nonzero whole number is evenly divisible by two if (and only if) its rightmost digit is divisible by 2. For example, check to see if 656 is evenly divisible by two.

$$6 / 2 = 3 \quad \text{Therefore 656 is divisible by two.}$$

Use this method to see if the following numbers are evenly divisible by 2.

- a) 153 b) 284 c) 367
d) 898 e) 1,092

(The method above is generally true and is an example of a mathematical theorem.)

2. Another mathematician claims that a nonzero whole number is evenly divisible by 3 if (and only if) the sum of its digits is evenly divisible by 3. For example, check to see if 375 is evenly divisible by 3.

$$3 + 7 + 5 = 15 \quad 15 / 3 = 5 \quad \text{Therefore 375 is divisible by 3.}$$

Use this method to see if the following numbers are evenly divisible by 3.

- a) 243 b) 187 c) 834
d) 951 e) 6,402

(The method above is generally true and is an example of a mathematical theorem.)

3. Yet another mathematician says that a nonzero number is evenly divisible by four if (and only if) its rightmost two digits are evenly divisible by four. For example, see if 1,932 is evenly divisible by 4.

$$32 / 4 = 8 \quad \text{Therefore, 1,932 is evenly divisible by 4.}$$

Use this method to see if the following numbers are evenly divisible by 4.

- a) 362 b) 712 c) 834
d) 956 e) 6,402

(The method above is generally true and is an example of a mathematical theorem.)

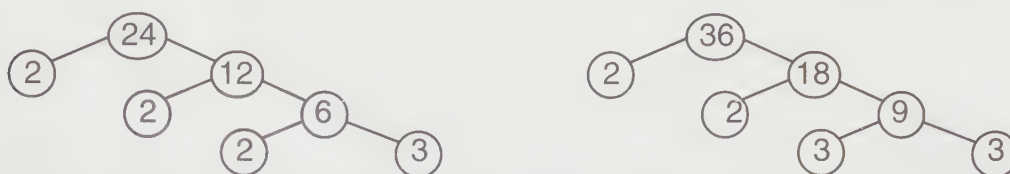
Name _____ Date _____

LESSON FOUR: Exercise 4**Greatest Common Factor**

The **greatest common factor** (sometimes called the **GCF**) of two or more nonzero whole numbers is the largest whole number that is a factor of all of the numbers.

An easy way to find the greatest common factor for two numbers is to find the prime factors for each of the numbers and then to identify all of the prime factors that are common to both numbers. The greatest common factor is just the product of the prime factors common to both numbers. As an example, we will find the GCF of 24 and 36.

The factor trees for 24 and 36 are as follows:



A list of the prime factors, from smallest to largest for each of the numbers is shown below. The ones for 24 are on the top line and the factors for 36 are on the bottom line.

Prime factors for 24	2	2	2	3
Prime factors for 36	2	2	3	3

The prime factors that both numbers have in common can be multiplied to find their greatest common factor.

$$\text{GCF} = 2 \times 2 \times 3 = 12$$

The greatest common factor can also be found for more than two numbers. For example, find the greatest common factor for 18, 24, 36.

The prime factors for each of these numbers are shown below and the ones in common are circled.

Prime factors for 18	2	3		3
Prime factors for 24	2	2	2	3
Prime factors for 36	2	2	3	3

The greatest common factor is the product of $2 \times 3 = 6$.

Name _____ Date _____

1. Find the greatest common factor for the following numbers.

a) 6 and 18

b) 27 and 24

c) 16 and 24

d) 30 and 45

e) 49 and 21

f) 48 and 32

g) 8, 48, and 92

h) 27, 36, and 81

i) 32, 48, and 64

Name _____ Date _____

2. There are 18 boys and 24 girls in a summer recreation program at a city park. The program director wants to divide them into groups of equal size to do a cleanup patrol. Each group, however, must be composed of all boys or all girls. What is the largest group that he could select? How many groups of that size would be all boys? How many groups of that size would be all girls?
3. A candy seller has 64 candy bars and 72 lollipops. He wants to place the candy in bags with either all candy bars or all lollipops in a bag. Each bag must contain the same number of items. What is the largest number of items that he can place in each bag? How many bags of candy bars will he have? How many bags of lollipops will he have?
4. Mrs. Ramirez is having a big dinner for her daughter's wedding reception. She finds that she must invite 54 of her daughter's friends and 63 of her daughter's new husband's friends. She feels that she should seat people at tables so that there are the same number of people at each table. She also feels that, at each table, she should seat only her daughter's friends or only her son-in-law's friends. What is the largest number of people that she can seat at each table? How many tables will she need?
5. Glinnis is in charge of a camping trip for some foreign exchange students. There are 12 British students who speak only English, eight French students who speak only French, and 16 German students who speak only German. She wants to divide them into hiking groups so that there are the same number of students in each group and so that everyone in each group speaks the same language. What is the largest number of people that she can place in each hiking group? How many total groups will there be?
6. Boy Scout Troop 27 has collected 45 cans of peas, 50 cans of beans, and 30 cans of corn in a food drive for needy persons. The troop leader needs to place all of the cans in boxes. All of the boxes must contain the same number of cans and each box can only contain cans of peas, cans of beans, or cans of corn. What is the largest number of cans that can be placed in each box? How many boxes will the leader need?

Name _____ Date _____

LESSON FOUR: Exercise 5**Least Common Multiple**

The **least common multiple** (sometimes called the **LCM**) of two or more nonzero whole numbers is the smallest nonzero whole number that is a multiple of all the numbers.

One way to find the LCM of two numbers is simply to list their multiples and then identify the number that is the smallest multiple of both of them. The example below shows how this can be done.

Find the LCM of 15 and 21

For 15:	$1 \times 15 = 15$	For 21:	$1 \times 21 = 21$
	$2 \times 15 = 30$		$2 \times 21 = 42$
	$3 \times 15 = 45$		$3 \times 21 = 63$
	$4 \times 15 = 60$		$4 \times 21 = 84$
	$5 \times 15 = 75$		$5 \times 21 = 105$
	$6 \times 15 = 90$		$6 \times 21 = 126$
	$7 \times 15 = 105$		$7 \times 21 = 147$

If you look carefully, the number 105 is the smallest number that appears in both columns (as 7×15 and 5×21 .) Therefore, 105 is the LCM of 15 and 21.

This technique can also be applied to more than two numbers.

Find the LCM of 4, 6, and 9

For 4:	$1 \times 4 = 4$	For 6:	$1 \times 6 = 6$	For 9:	$1 \times 9 = 9$
	$2 \times 4 = 8$		$2 \times 6 = 12$		$2 \times 9 = 18$
	$3 \times 4 = 12$		$3 \times 6 = 18$		$3 \times 9 = 27$
	$4 \times 4 = 16$		$4 \times 6 = 24$		$4 \times 9 = 36$
	$5 \times 4 = 20$		$5 \times 6 = 30$		$5 \times 9 = 45$
	$6 \times 4 = 24$		$6 \times 6 = 36$		$6 \times 9 = 54$
	$7 \times 4 = 28$		$7 \times 6 = 42$		$7 \times 9 = 63$
	$8 \times 4 = 32$		$8 \times 6 = 48$		$8 \times 9 = 72$
	$9 \times 4 = 36$		$9 \times 6 = 54$		$9 \times 9 = 81$

The number 36 is the smallest number that appears in all three columns. Therefore 36 is the LCM of 4, 6, and 9.

Name _____ Date _____

1. Find the least common multiple of the following numbers.

a) 6 and 9

b) 9 and 5

c) 16 and 24

d) 30 and 45

e) 12 and 21

f) 17 and 32

g) 2, 5, and 8

h) 3, 7, and 9

i) 6, 12, and 21

Name _____ Date _____

2. Millicent works for an ice cream company that sells ice cream to restaurants in two different sizes, a seven-quart container and a five-quart container. When a batch of ice cream is made, it must be used to fill a group of seven-quart containers, or a group of five-quart containers. What is the smallest batch (in quarts) that can be made to fill a whole number of seven-quart containers or a whole number of five-quart containers with no ice cream left over?
3. An army officer sends squads of six soldiers on short-range night patrols and squads of 10 soldiers on long-range night patrols. On a given night, she either assigns all of her soldiers to short-range patrols, or she assigns all of them to long-range patrols. What is the smallest number of soldiers that she can have in her command that will staff a whole number of short-range patrols or a whole number of long-range patrols, with no soldiers left over?
4. School bus A gets five miles per gallon of gas and school bus B gets 11 miles per gallon of gas. Both buses carried students from their hometown school to an out-of-town school for a football game. Both buses used a whole number of gallons of gasoline. What is the smallest possible distance to the out-of-town school?
5. A mountain man is building a cabin. He wants to use leftover logs from a nearby sawmill to build the side walls of the cabin. He must use a whole number of logs for the length of the side wall. Sometimes the sawmill has six-foot leftover logs, sometimes it has eight-foot ones, and sometimes it has 12-foot ones. What is the shortest side wall that he can have on his cabin that could be made with a whole number of six-foot, eight-foot, and 12-foot logs?
6. A farmer in an Asian country grows rice. Every 10 days a rice buyer comes by and purchases rice. The rice buyer brings all three-bushel containers, all five-bushel containers, or all seven-bushel containers with him. The farmer, however, never knows which size of container the buyer will have until the buyer arrives. What is the smallest number of bushels of rice that the farmer should have ready so that he can fill a whole number of three-bushel, five-bushel, or seven-bushel containers?

Name _____ Date _____

LESSON FIVE: ROOTS AND EXPONENTIAL PROBLEMS

In this lesson, we will look at several types of problems that use numbers to powers and roots. We will also investigate scientific notation, a method that is used to represent very large numbers. Finally, we will look at some properties of real numbers and decimals.

Roots

Many mathematical problems require that you find **roots** of numbers. For example, finding the number that when squared (multiplied times itself) gives 9 is called finding the **square root** of 9. As another example, finding the number that when cubed (ex. $2 \times 2 \times 2 = 8$) gives 8 is called finding the **cube root** of 8. As we will see, many other roots, such as the **fourth root** and the **sixth root**, are possible.

Finding the root of a number is indicated by **the radical**, $\sqrt{\quad}$. If there is a number above the $\sqrt{\quad}$ in the radical, it indicates the root to be found. This number is called the **index**. The number inside the radical is called the **radicand**. If there is no index, the root is assumed to be a square root. Some examples of radicals are shown below.

$$\sqrt[4]{16} = 2 \quad \sqrt{25} = 5 \quad \sqrt[3]{27} = 3 \quad \sqrt[6]{64} = 2 \quad \sqrt[4]{81} = 3 \quad \sqrt[5]{1,024} = 4$$

Finding the root of a number can sometimes be thought of as finding two or more equal factors of that number. The factor raised to the appropriate power gives you the original number.

$$\sqrt[4]{16} = 2$$

Sixteen can be thought of as having 4 factors equal to 2.

$$2 \times 2 \times 2 \times 2 = 2^4 = 16$$

Negative roots

It is possible for a root of an integer to be negative. In fact, the square root, fourth root, or other even index root of an integer can be a positive or a negative number.

$$\sqrt{4} = 2 \quad \text{or} \quad \sqrt{4} = -2$$

$$\text{because } 2 \times 2 = 4 \quad \text{or} \quad -2 \times -2 = 4$$

It is not possible, however, to find an even index root of a negative number. (The result of such an attempt will give you an imaginary number, something that you will eventually study in an algebra class.)

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It is possible for a negative number to have an odd index root, such as a cube root or a fifth root.

$$\sqrt[3]{-8} = -2 \quad \text{because} \quad -2 \times -2 \times -2 = -8$$

$$\sqrt[5]{-32} = -2 \quad \text{because} \quad -2 \times -2 \times -2 \times -2 \times -2 = -32$$

Adding and Subtracting Radicals

Sometimes you will see radicals multiplied by whole numbers or integers. These numbers are written in the forms shown below.

$$2\sqrt[3]{8}$$

$$5\sqrt{9}$$

$$3\sqrt[3]{125}$$

$$6\sqrt{3}$$

If the radicand of the radical is a number whose root can be easily found, the product of a number and a radical can be simplified to a single number.

$$5\sqrt{9} = 5 \times 3 = 15$$

In other cases, the number is just left as a whole number times a radical. $6\sqrt{3}$ is an example.

Sometimes it is necessary to add or subtract radicals or numbers multiplied times radicals. Additions and subtractions can only be done if the radicals involved have the same index and the same radicand.

$$5\sqrt{5} + 3\sqrt{5} = (5 + 3)\sqrt{5} = 8\sqrt{5}$$

$$6\sqrt{7} - 2\sqrt{7} = (6 - 2)\sqrt{7} = 4\sqrt{7}$$

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LESSON FIVE: Exercise 1

1. Find the indicated roots of the following whole numbers.

a) $\sqrt{36} =$

b) $\sqrt{81} =$

c) $\sqrt{49} =$

d) $\sqrt[4]{81} =$

e) $\sqrt{64} =$

f) $\sqrt[3]{64} =$

g) $\sqrt[3]{125} =$

h) $\sqrt[5]{243} =$

i) $\sqrt[4]{625} =$

j) $\sqrt[6]{729} =$

k) $\sqrt[5]{3,125} =$

l) $\sqrt[4]{2,401} =$

2. The square root of a number is 11. What is the number?

3. The square root of a number is 13. What is the number?

4. The cube root of a number is 5. What is the number?

5. The cube root of a number is 9. What is the number?

6. The fourth root of a number is 3. What is the number?

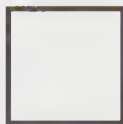
7. The fifth root of a number is 7. What is the number?

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LESSON FIVE: Exercise 2

1. The area of a square, whose sides are measured in inches, can be thought of as the number of one-inch by one-inch squares that could be placed inside the square. The area is expressed in square inches. If you know the area of a square, you can find the lengths of its sides (the lengths of its two sides are equal) by finding the square root of its area.

For example, suppose the area of a square is 9 square inches. What are the lengths of its sides?



$$\text{side length} = \sqrt{9} = 3 \text{ inches}$$

- a) The area of a square is 16 square inches. What are the lengths of the sides of the square in inches?
- b) The area of a square is 49 square inches. What are the lengths of the sides of the square in inches?
- c) The area of a square is 144 square inches. What are the lengths of the sides of the square in inches?

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2. The volume of a cube, whose edges are measured in inches, can be thought of as the number of one-inch by one-inch by one-inch small cubes that could be placed inside the larger cube. You can find the lengths of its edges (the lengths of all of its edges are equal) by finding the cube root of its volume.

For example, suppose the volume of a cube is 8 cubic inches. What are the lengths of its edges?



$$\text{Edge length} = \sqrt[3]{8} = 2 \text{ inches}$$

- a) A cube has a volume of 27 cubic inches. What are the lengths of the edges of the cube?
- b) A cube has a volume of 125 cubic inches. What are the lengths of the edges of the cube?
- c) A cube has a volume of 2,197 cubic inches. What are the lengths of the edges of the cube?
3. Deanna was teasing her cousin, Tara. "I know this cute boy who likes you," she said, "But I won't tell you his age. However, I'll give you a hint. His age is the sum of the cube root of your grandmother's age plus the square root of your younger brother's age." Tara knew that her younger brother was 9 years old, and after some tactful questions, she found out that her grandmother's age was 64. How old was the boy that liked Tara?

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4. A number is one more than another number. The square of the first number is one more than the cube of the second number. What are the two numbers?

5. Find the negative roots of the following numbers.

a) $\sqrt{64} =$

b) $\sqrt{81} =$

c) $\sqrt[3]{-64} =$

d) $\sqrt[3]{-27} =$

e) $\sqrt[4]{16} =$

f) $\sqrt[5]{-243} =$

6. Find the sums and differences of the following numbers that involve radicals. Simplify your answers to whole numbers or integers, write them with radicals that contain the smallest possible radicands.

a) $7\sqrt{4} + 5\sqrt{4} =$

b) $21\sqrt[3]{27} + 5\sqrt[3]{27} =$

c) $10\sqrt[4]{16} + 15\sqrt[4]{16} =$

d) $4\sqrt{5} + 13\sqrt{5} =$

e) $9\sqrt{36} - 2\sqrt{36} =$

f) $14\sqrt[3]{64} - 5\sqrt[3]{64} =$

g) $29\sqrt{121} =$

h) $7\sqrt{7} - 3\sqrt{7} =$

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LESSON FIVE: Exercise 3

1. The Pythagorean theorem is an interesting relation that applies to right triangles. A right triangle is a three-sided geometric figure, like the one shown below, in which two of the sides are perpendicular.



The long side of such a triangle is called the **hypotenuse**. The length of the hypotenuse can be found if the lengths of the other two sides are known. It is the square root of the sum of the squares of the two shorter sides. A common problem in math courses such as geometry and trigonometry is to calculate the hypotenuse of a right triangle. The following example shows how this can be done.

The two shorter sides of a right triangle are three inches and four inches long. Find the length of the hypotenuse in inches.

$$\text{hypotenuse} = 3^2 + 4^2 = 9 + 16 = \sqrt{25} = 5$$

- a) The two shorter sides of a right triangle are 12 and 5 inches long. What is the length of the hypotenuse in inches?
- b) The two shorter sides of a right triangle are 12 and 16 inches long. Find the length of the hypotenuse in inches.
- c) The two shorter sides of a right triangle are 48 and 20 inches long. Find the length of the hypotenuse in inches.

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LESSON FIVE: Exercise 4**Multiplying and Dividing Exponential Numbers**

In lesson three, we looked at exponential numbers and found that they could be used to solve some interesting problems. There are some operations that can be performed with exponential numbers that allow them to be used in more problems.

Exponential numbers can be multiplied if they have the same base. The resulting product is just the base to the sum of the powers of the exponentials that are being multiplied.

$$3^2 \times 3^3 = 3^{2+3} = 3^5 = 243$$

If the exponential numbers in the product are multiplied by other numbers, the product of the other numbers is found and multiplied times the product of the exponentials.

$$5 \times 2^2 \times 7 \times 2^3 = 5 \times 7 \times 2^{2+3} = 35 \times 2^5 = 35 \times 32 = 1,120$$

Division of exponential numbers is similar to multiplication, except that the power of the divisor is subtracted from the power of the dividend. The exponential numbers that are being divided must have the same base.

$$\frac{4^5}{4^2} = 4^{5-2} = 4^3 = 64$$

If the exponential numbers in the quotient are multiplied by other numbers, the quotient of the other numbers is found and multiplied times the quotient of the exponentials.

$$\frac{8 \times 3^7}{2 \times 3^4} = (8/2) \times 3^{7-4} = 4 \times 3^3 = 4 \times 27 = 108$$

Exponential numbers can also be taken to a power. The result is the base to the product of the powers that are involved.

$$(3^2)^3 = 3^{2 \times 3} = 3^6 = 729$$

Multiplying and Dividing Radicals

The rules for multiplying and dividing radicals are much like those for multiplying and dividing exponential numbers. In fact, as you will see in algebra classes, a square root of a number is just the number to the $\frac{1}{2}$ power, a cube root of a number is just the number to the $\frac{1}{3}$ power, and so on.

Radicals may be multiplied and divided if they have the same index, even if they have different radicands. The following examples show how multiplication and division are performed.

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$$\sqrt{4} \times \sqrt{9} = \sqrt{4 \times 9} = \sqrt{36} = 6$$

$$2 \sqrt[3]{27} \times 5 \sqrt[3]{8} = (2 \times 5) \sqrt[3]{27 \times 8} = 10 \sqrt[3]{216} = 10 \times 6 = 60$$

$$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{12/3} = \sqrt{4} = 2$$

$$\frac{8\sqrt{18}}{2\sqrt{2}} = (8/2)\sqrt{18/2} = 4\sqrt{9} = 4 \times 3 = 12$$

Radicals may also be raised to a power. If the power is equal to a multiple of the index of the radical, the radical will be removed and the radicand raised to a new power. The new power is the original power divided by the index of the radical. (You will learn more about raising radicals to powers in your algebra class when you study fractional powers.)

$$(\sqrt[4]{5})^8 = 5^{8/4} = 5^2 = 25$$

1. Find the following products, quotients, and powers of exponential numbers.

a) $2^2 \times 2^3 =$

b) $5^3 \times 2^2 =$

c) $3^4 \times 3^3 =$

d) $5 \times 3^2 \times 4 \times 3^1 =$

e) $7 \times 2^2 \times 5 \times 2^3 =$

f) $9 \times 5^3 \times 11 \times 5^4 =$

g) $\frac{2^9}{2^5} =$

h) $\frac{5^7}{5^4} =$

i) $\frac{8^6}{8^4} =$

j) $\frac{8 \times 2^5}{4 \times 2^2} =$

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k) $\frac{14 \times 5^7}{2 \times 5^4} =$

l) $\frac{15 \times 7^9}{3 \times 7^7} =$

m) $(2^3)^4 =$

n) $(5^3)^2 =$

o) $(6^2)^2 =$

2. Mr. Martinez was a math teacher who liked to give his students quick thought problems. He would not tell the students how old he was, but he gave them the following clue.

His age, he said, could be found by finding the correct whole number. Four times the square of this whole number plus three times the cube of one more than the whole number would give his age. He also told them that he was 23 when he graduated, and he was too young to retire. The retirement age at his school was 65. How old is Mr. Martinez?

3. Peg was watching a science and math show on TV. There was an Internet site that had some more information about the show. The host of the show said that the password to the Internet site was a whole number that was equal to two times the cube of the day of the month plus three times the square of the day of the month. Peg checked the calendar and found that the date was April 3. What number should she use for the Internet password?
4. (See problem 3.) A week later, the host of the TV show explained that the password to the Internet site was now 121 times the cube of the date divided by the square of one day more than the date. What number should Peg now use for the Internet password?

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LESSON FIVE: Exercise 5

1. Find the following products, quotients, and powers of radicals.

a) $\sqrt{20} \times \sqrt{5} =$

b) $\sqrt[3]{16} \times \sqrt[3]{4} =$

c) $\sqrt{2} \times \sqrt{3} =$

d) $5 \times \sqrt[3]{8} \times 4 \times \sqrt[3]{8} =$

e) $7 \times \sqrt{4} \times 5 \times \sqrt{9} =$

f) $9 \times \sqrt{5} \times 11 \times \sqrt{5} =$

g) $\frac{\sqrt{72}}{\sqrt{2}} =$

h) $\frac{\sqrt[4]{3125}}{\sqrt[4]{5}} =$

i) $\frac{\sqrt[3]{12}}{\sqrt[3]{4}} =$

j) $\frac{8 \times \sqrt[3]{16}}{4 \times \sqrt[3]{2}} =$

k) $\frac{14 \times \sqrt[4]{80}}{2 \times \sqrt[4]{5}} =$

l) $\frac{15 \times \sqrt{28}}{3 \times \sqrt{7}} =$

m) $(\sqrt{3})^4 =$

n) $(\sqrt[3]{5})^3 =$

o) $(\sqrt[4]{2})^{12} =$

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2. Candi was pouring water into her empty fish tank. She was using a strange jar to carry water from the kitchen sink to the fish tank. It held $\sqrt{2}$ gallons of water. She poured four jars of water into the fish tank and was then interrupted by a phone call. When she came back, she added three more jars of water. How much water did she pour into the tank? (Leave your answer as a multiple of a square root.)

3. (Refer to problem 2.) Candi noticed that she had too much water in her tank, so she dipped out two jars full and poured water into the kitchen sink. How much water is now in the tank? (Leave your answer as a multiple of a square root.)

4. Sometimes the lengths of sides of geometrical figures can be written as square roots or cube roots of other numbers. Suppose that the length of a rectangle is $\sqrt{9}$ inches and the width of the rectangle was $\sqrt{4}$ inches. The area of a rectangle is its length times its width, and it is expressed in square inches. What is the area of this rectangle in square inches?

5. Sometimes the lengths of sides of geometrical figures can be written as square roots or cube roots of other numbers. Suppose that the edge of a cube was $\sqrt[3]{7}$ inches. (All of the edges of a cube are equal length.) The volume of a cube is just the cube of the length of one of its edges, and it is expressed in cubic inches. What is the volume of this cube in cubic inches?

6. A length of two-by-four board is $8\sqrt{12}$ feet long. The length of another two-by-four board is $2\sqrt{2}$ feet. How many times as long is the first board as the second one?

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LESSON FIVE: Exercise 6**Real Numbers**

In the last four lessons, we have looked at problems that involve whole numbers and integers in relatively simple forms. There are many problems, however, that force us to use real numbers. **Real numbers** are numbers that can have a **decimal point** and a **fractional part**. You will explore many of their properties when you take classes in algebra, but we will introduce some applications that involve them in this lesson and the next one.

The fractional part of a real number is often described by the number of decimal places that are being used. A **decimal place** is just a digit that is to the right of the decimal point. For example, consider the two real numbers below.

15.8

17.25

The first number has one decimal place because there is only one digit to the right of the decimal point. The second number has two decimal places because there are two numbers to the right of the decimal point. Most of the real number examples and problems in this book will have one or two decimal places.

The decimal parts of real numbers describe fractional parts of numbers. If there is one decimal place, the fractional part is found by dividing the decimal digit by 10 and then simplifying.

$$15.8 = 15 \text{ and } \frac{8}{10} = 15 \text{ and } \frac{4}{5}$$

If there are two decimal places, the fractional part is found by dividing the two decimal digits by 100 and then simplifying.

$$17.25 = 17 \text{ and } \frac{25}{100} = 17 \text{ and } \frac{1}{4}$$

It is just as easy to convert fractions in lowest terms to decimals. The following examples show how this can be done. Convert the simplified fraction to a number over tenths or hundredths, and then divide the numerator by the denominator to get the decimal.

$$234 \text{ and } \frac{2}{5} = 234 \text{ and } \frac{4}{10} = 234.4$$

$$68 \text{ and } \frac{3}{4} = 68 \text{ and } \frac{75}{100} = 68.75$$

When real numbers are used in calculations, it is often easiest to add, subtract, multiply, and divide them using a calculator. If your teacher thinks you should use a calculator for real number problems, he or she will show you how to do real number math with it.

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1. Convert the following numbers with fractional parts to decimal numbers.

a) $12 \text{ and } \frac{1}{2} =$

b) $24 \text{ and } \frac{1}{4} =$

c) $85 \text{ and } \frac{4}{5} =$

d) $256 \text{ and } \frac{9}{10} =$

e) $305 \text{ and } \frac{3}{4} =$

f) $42 \text{ and } \frac{2}{5} =$

2. Convert the following decimal numbers to numbers with fractional parts.

a) $25.5 =$

b) $14.25 =$

c) $55.75 =$

d) $256.2 =$

e) $305.75 =$

f) $42.9 =$

3. A length of two-by-four board is 11.25 feet. The length of another two-by-four board is 4.5 feet. How many times as long is the first board as the second one?

4. A wooden block is 2.25 inches thick. Another wooden block is 5.5 inches thick. One block is placed on a tabletop, and the second block is stacked on top of the first one. How high above the tabletop is the surface of the second block?

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5. Six identical blocks are 1.75 inches thick. The six blocks are stacked one on top of the other on a tabletop. How much higher is the stack of blocks than the tabletop?
6. Chip is 72.75 inches tall and Muffy is 65.5 inches tall. How much taller is Chip than Muffy?
7. Quarettie is following a recipe for making a big batch of butterscotch pudding. She first uses a jar that holds 1.25 pints and adds two of these jars full of milk to her mixing bowl. Then she switches to a jar that holds 2.5 pints and adds three jars full of milk to the mixing bowl. How many pints of milk does Quarettie add to the mixing bowl?
8. The area of a rectangle is given by its length in inches times its width in inches and is expressed in square inches. What is the area of a rectangle whose length is 8.5 inches and whose width is 4.5 inches?
9. There are 60 minutes in an hour. Write 2.5 hours as 2 hours and some number of minutes.
10. (Refer to problem 9.) Write 4.4 hours as 4 hours and some number of minutes.
11. (Refer to problems 9 and 10.) Write 6 hours 45 minutes as some number of hours with two decimal places.

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LESSON FIVE: Exercise 7**Scientific Notation**

Very large or very small numbers are sometimes expressed in scientific notation. Scientific notation is a convenient way of expressing the numbers so that they do not contain large numbers of zeros. In this lesson, we will only be concerned with putting very large numbers into scientific notation.

Scientific notation is based on powers of 10. Ten to the zero (10^0) power is 1, 10 to the one power (10^1) is 10, 10 to the two power (10^2) is 100, and so on. A large number can be placed in scientific notation by writing its left-most nonzero digit, followed by a decimal point, followed by the other digits in the number, all multiplied by a positive power of 10. The numbers below are written in scientific notation.

$$3.79 \times 10^{12}$$

$$8.1 \times 10^5$$

To convert a large number to scientific notation, you must do two things. First, you need to move the decimal point in the number to the left so that it is between the left most nonzero digit in the number and the next digit to it on the right. Second, you need to multiply a positive power of 10. The power should be equal to the number of digits that you passed over when you moved the decimal point.

The following example shows how to convert a large number to scientific notation.

$$5,730,000,000$$

First, move the decimal point (assumed to be at the right-hand end of the number) to a position between 5 and 7. You will need to move it nine places to the left. Then multiply by 10^9 . (The power of 10 is 9 because you moved the decimal point nine places, and it is positive because you moved it to the left.) You can usually drop off all of the trailing zeros. The result is shown below.

$$5.73 \times 10^9$$

Numbers in scientific notation can be converted back into their ordinary representations by simply undoing the steps that would have converted them to scientific notation.

Adding and Subtracting Numbers in Scientific Notation

Numbers in scientific notation can be added and subtracted, but only if their powers of 10 are the same. Therefore, when you want to add or subtract two such numbers, you must adjust the power of 10 of one or both of them. You can do this by increasing or decreasing the power by the appropriate amount and then moving the decimal point to the right or the left by a number of places equal to the increase or decrease in the power. This will insure

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that the value of the number will remain unchanged. For example, if you increased the power of 10 by two, you would need to move the decimal point two places to the left. If you decreased the power of 10 by three, you would need to move the decimal point three places to the right.

The following example shows how two numbers in scientific notation can be added.

$$3.25 \times 10^5 + 4.62 \times 10^3$$

To get the same power of 10 for each number, the first one will be converted to $\times 10^3$. Since the power of 10 is decreased by two, the decimal point will have to be moved two places to the right.

$$\begin{aligned} 325.0 \times 10^3 + 4.62 \times 10^3 &= 329.62 \times 10^3 \\ &= 3.2962 \times 10^5 \end{aligned}$$

Often, we round off numbers like the one above to two decimal places.

$$= 3.30 \times 10^5$$

Multiplying and Dividing Numbers in Scientific Notation

Numbers in scientific notation can also be multiplied and divided. The process is very simple. To multiply two numbers, multiply their number portions and add their powers of 10. Consider the example below.

$$\begin{aligned} (2.3 \times 10^3) \times (3.5 \times 10^4) &= (2.3) \times (3.5) \times 10^{(3+4)} \\ &= 8.05 \times 10^7 \end{aligned}$$

Dividing numbers in scientific notation is just as easy. To divide two numbers, divide the number portion of the dividend by the number portion of the divisor and subtract the power of the divisor from the power of the dividend. Consider the example below.

$$\frac{8.75 \times 10^6}{2.5 \times 10^2} = \frac{8.75}{2.5} \times 10^{(6-2)} = 3.5 \times 10^4$$

1. Convert the following large numbers to scientific notation.

a) 270 =

b) 5,600 =

c) 987,000 =

d) 318,000,000 =

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2. Convert the following large numbers from scientific notation to ordinary numbers.

a) $2.5 \times 10^2 =$

b) $5.7 \times 10^3 =$

c) $9.22 \times 10^5 =$

d) $1.23 \times 10^8 =$

3. Add and subtract the following numbers in scientific notation.

a) $5.27 \times 10^5 + 2.0 \times 10^3 =$

b) $1.24 \times 10^7 + 4.6 \times 10^6 =$

c) $4.22 \times 10^3 - 2.1 \times 10^2 =$

d) $6.77 \times 10^4 - 4.0 \times 10^2 =$

4. Multiply and divide the following numbers in scientific notation.

a) $(1.5 \times 10^3) \times (3.5 \times 10^2) =$

b) $(2.0 \times 10^4) \times (5.7 \times 10^1) =$

c) $(7.5 \times 10^7) \times (5.2 \times 10^8) =$

d) $\frac{1.4 \times 10^5}{7 \times 10^2} =$

e) $\frac{5.1 \times 10^7}{4.25 \times 10^3} =$

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LESSON SIX: ELEMENTARY EQUATION PROBLEMS**LESSON SIX: Exercise 1**

One of the most important steps that you will take in your first algebra class is to use symbols to represent numeric quantities. In some cases, the symbols will be **unknown** quantities in equations. Your task will be to find the value of some unknown that is represented by a symbol. In other cases, however, the symbols will be used in formulas. A **formula** is just a shorthand notation that shows how a value can be calculated.

Formulas

The formulas that we will look at will all have pretty much the same form. They will consist of a symbol, an equal sign, and other symbols separated by arithmetic operators, such as plus signs or multiply signs. The example below shows a typical formula.

$$A = B \times C + D$$

This formula is composed of four symbols, an **A**, a **B**, a **C**, and a **D**. The **A** on the left-hand side of the equation stands for a quantity that you want to calculate. The **B**, **C**, and **D** stand for values that you must have to calculate **A**. The formula is like a recipe for making **A**. If you can get values for **B**, **C**, and **D**, it shows you how to get **A**. For example, suppose **B** was equal to **3**, **C** was equal to **5**, and **D** was equal to **2**. You could calculate **A** by substituting the other values for their letters in the formula.

$$\begin{aligned} A &= B \times C + D \\ &= 3 \times 5 + 2 \\ &= 15 + 2 \\ &= 17 \end{aligned}$$

In the problems that follow, you will be asked to use formulas to calculate various quantities. The first ones will be very simple, but some of the latter ones will be pretty complicated. All of them, however, will give you practice in thinking with symbols. (Use your calculator on the problems that involve real numbers, if your teacher says that you may do so.)

1. Find the value of **A** in the following formulas.

a) $A = B + C + D$ $B = 3, C = 4, D = 5$

$A = \underline{\hspace{2cm}}$

b) $A = D - C/B$ $B = 3, C = 6, D = 5$

$A = \underline{\hspace{2cm}}$

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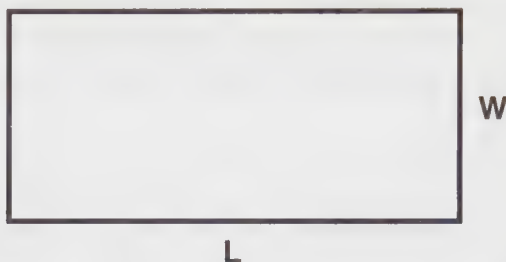
c) $A = B/(C + D)$ $B = 21, C = 4, D = 3$

$A = \underline{\hspace{2cm}}$

d) $A = B * C + D^2$ $B = 3, C = 4, D = 2$

$A = \underline{\hspace{2cm}}$

2. In previous lessons, you have solved several problems that involve area. Area is a measure of the amount of surface occupied by a two-dimensional figure. It is expressed in units of length squared, sometimes called square units. To see why area must be described in square units, look at the rectangle below. It has length, **L**, and width, **W**.



The area of the rectangle is given by its length times its width.

$$\text{Area} = L \times W$$

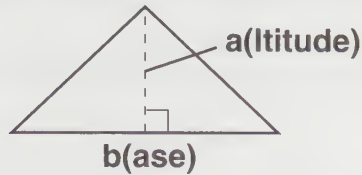
This results in two quantities with the units of length being multiplied together. The resulting unit of area would be length to the second power or length squared.

- a) A rectangle has a length of five inches and a width of four inches. Use the formula to compute the area of the rectangle.
- b) A rectangle has a length of 12 inches and a width of nine inches. Use the formula to compute the area of the rectangle.
- c) A rectangle has a length of 25 feet and a width of 16 feet. Use the formula to compute the area of the rectangle.

Name _____ Date _____

LESSON SIX: Exercise 2

1. A three-sided figure, called a triangle, is shown in the picture below.



It's area can be computed from the two quantities shown in the picture, the base **b** and the altitude **a**.

$$\text{Area} = \frac{1}{2} \times b \times a$$

- a) Use the formula to find the area of a triangle whose base is six inches and whose altitude is four inches.
- b) Use the formula to find the area of a triangle whose base is three feet and whose altitude is four feet.
- c) Use the formula to find the area of a triangle whose base is 11 feet and whose altitude is 14 feet.

2. The area of a circle is given by the formula,

$$\text{Area} = \pi r^2$$

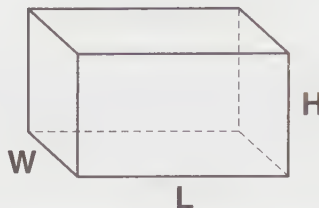
Where π is the constant number 3.1416... and **r** is the radius of the circle. The radius is expressed in length units and the area is given in square units.

- a) Use the formula to find the area, in square inches, of a circle whose radius is two inches.
- b) Use the formula to find the area, in square inches, of a circle whose radius is nine inches.
- c) Use the formula to find the area, in square feet, of a circle whose radius is five feet.

Name _____ Date _____

LESSON SIX: Exercise 3

1. In previous lessons you have worked several problems that involve volume. Volume is a measure of the amount of space occupied by a body. It is expressed in units of length cubed, sometimes called cubic units. To see why volume must be described in cubic units, consider the rectangular box below. It has length, **L**, width, **W**, and height, **H**.



The volume of the box is given by its length times its width times its height.

$$\text{Volume} = L \times W \times H$$

This results in three quantities with units of length being multiplied together. The resulting unit of volume would be length to the third power or length cubed.

- a) A rectangular box has a length of five inches, a width of four inches, and a height of three inches. Use the formula to compute the volume of the rectangular box.
 - b) A rectangular box has a length of 12 inches, a width of nine inches, and a height of eight inches. Use the formula to compute the volume of the box.
 - c) A box has a length of 25 feet, a width of 16 feet, and a height of seven feet. Use the formula to compute the volume of the box.
2. A sphere is a geometric figure in the shape of a ball. The volume of a sphere is found by using the following formula.

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Where **r** is the radius of the sphere, the radius is expressed in length units and the volume in cubic length units.

- a) Use the formula to find the volume, in cubic inches, of a sphere whose radius is two inches.
- b) Use the formula to find the volume, in cubic inches, of a sphere whose radius is four inches.

Name _____ Date _____

LESSON SIX: Exercise 4

1. The average speed of a body is a measure of how fast the body is traveling during some period of time. It is usually expressed in length units divided by time units. Feet per second or miles per hour are common measures of average speed. A formula for finding the speed of a body is given below, in words and in symbols.

$$\text{Speed} = \frac{\text{Distance traveled}}{\text{Time of travel}}$$

$$v = \frac{d}{t}$$

The symbol **v** stands for speed, **d** is the symbol for the distance traveled, and the symbol for the time it took to travel the distance is **t**.

- a) A car travels 60 miles in two hours. What is its average speed in miles per hour?
 - b) A man runs 100 feet in five seconds. What is his average speed in feet per second?
 - c) A spaceship travels 5.5×10^5 miles in 2.5×10^2 hours. What is its average speed in miles per hour?
2. The formula in problem #1 can be rearranged to calculate the distance traveled if a body moves at a given average speed for a certain amount of time.

$$d = v \times t$$

The distance traveled is **d**, **v** is the average speed, and the time the body travels is **t**.

- a) A car travels at 40 miles per hour for three hours. How many miles does it travel?
- b) A man runs at 15 feet per second for 10 seconds. How many feet does he run?
- c) A spaceship travels at 3,000 miles per hour for 20 hours. How many miles does it travel?

Name _____ Date _____

3. The formulas from problems #1 and #2 can be rearranged again to calculate the time that it would take to travel a given distance at a specified average speed.

$$t = \frac{d}{v}$$

The symbol for distance traveled is **d**, **v** is the symbol that stands for the speed, and **t** is the symbol for the time that it took the body to travel the distance.

- a) A car travels 120 miles at 40 miles per hour. How many hours does it travel?
- b) A man runs 450 feet at 15 feet per second. How many seconds does it take?
- c) Light travels at 1.86×10^5 miles per second. It is approximately 9.3×10^7 miles from the Sun to the earth. How many seconds does it take light to travel from the Sun to Earth?
4. The pressure of gas is measured in pounds per square inch. A pound is a measure of force. A force can be thought of as a push on something. In this case it is a measure of how much the gas is pushing on the walls of a container, the pressure in pounds per square inch of the gas container. If the temperature of the gas is kept constant, the product of the pressure times the volume (in cubic inches or some other volume unit) of the gas container is a constant number.

$$P \times V = \text{some number}$$

P here is the pressure of the gas in pounds per square inch, and **V** is its volume in cubic inches. The formula can also be expressed in the following two forms.

$$P = \frac{\text{some number}}{V}$$

- a) For a certain gas, **P x V = 20**. The pressure of the gas is 10 pounds per square inch. What is the volume occupied by the gas?
- b) For a certain gas, **P x V = 35**. The pressure of the gas is seven pounds per square inch. What is the volume occupied by the gas?
- c) For a certain gas, **P x V = 45**. The pressure of the gas is nine pounds per square inch. What is the volume occupied by the gas?

Name _____ Date _____

LESSON SIX: Exercise 5

1. The **factorial** of a number is given a strange-looking formula, the number with an exclamation mark after it. It is calculated as the same number times the number minus one times the number minus two and so on down to times one.

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$3! = 3 \times 2 \times 1 = 6$$

$$1! = 1$$

Zero is given the special factorial value of one.

$$0! = 1$$

- a) What is 2!? _____
- b) What is 6!? _____
- c) What is 8!? _____
2. When you invest money in a bank, the bank pays you some extra money called **interest** for letting them use your money. This interest is sometimes given as a certain percent per year. For example, the bank might pay you three percent of the money you invested if they use that money for a year.

If you leave your money in the bank for a long time, the money that you have increases in value at the end of each year. But, of course, at the end of each year you have more money so the increase is larger for each following year.

A formula that shows you how much money you would have at the end of some number of years is shown below.

$$A = D (1 + R/100)^Y$$

The **A** is the amount that you have after the **Y** years, **D** is the amount that you originally deposited, and **R** is the yearly interest rate in percent.

Name _____ Date _____

- a) Suppose you deposit \$100 in a bank that pays five percent interest per year. If you leave the money in the bank for five years, how much will you have at the end of that time?
- b) Suppose you deposit \$200 in a bank that pays four percent interest per year. If you leave the money in the bank for 10 years, how much will you have at the end of that time?
- c) Suppose you deposit \$1,000 in a bank that pays five percent interest per year. If you leave the money in the bank for 20 years, how much will you have at the end of that time?
3. The gravity on the surface of a planet is sometimes measured in **Gs**. One G is one earth gravity. Gravity in Gs for other planets can be computed from the following equation.

$$G = \frac{m}{r^2}$$

The **G** in this equation is the gravity in Gs, the **m** is the mass of the other planet expressed as a multiple of the earth's mass, and **r** is the radius of the other planet, expressed as a multiple of the earth's radius. The mass of something is a measure of how much physical material is in that something.

- a) A new planet is discovered at a nearby star. It has a mass of 20 earth masses and a radius of two earth radii. What would the gravity be in Gs on its surface?
- b) Another planet has a mass of 10 earth masses and a radius of five earth radii. What would the gravity be in Gs on its surface?

Name _____ Date _____

LESSON SIX: Exercise 6**Equations**

Mathematical problems often involve equations. An **equation** is a mathematical statement that says that two expressions are equal. The two expressions are usually on the left and right side of an equal sign. The equal sign indicates that they are to be equal.

In many cases, one or both of the expressions will involve a symbol that stands for an unknown quantity. This symbol is often just called the **unknown**. It will be your job to use the equation to determine the correct value for the unknown.

There are several mathematical operations that you can perform to simplify the equation to find the unknown. **In all cases, however, you must perform the same operation on both sides of the equation.**

Two operations that are often performed on equations are to add or subtract the same quantity to or from both sides. **You may add the same quantity to both sides of an equation or subtract the same quantity from both sides of an equation, and the two sides will remain equal.** The following example shows a simple equation involving an unknown whose symbol is x .

$$x + 6 = 11$$

This equation may be solved by subtracting 6 from both sides.

$$x + 6 - 6 = 11 - 6$$

$$x = 5$$

You may also multiply both sides of an equation by the same quantity or divide both sides of an equation by the same quantity and the two sides will remain equal. The example below shows a simple equation that is solved by dividing both sides by 5.

$$5x = 35$$

$$\frac{5x}{5} = \frac{35}{5}$$

$$x = 7$$

You may also switch the two expressions to the opposite sides of the equal sign, and the two sides will remain equal.

In some cases, you may need to apply several of the above operations to solve for the unknown.

Name _____ Date _____

1. Solve the following equations for the unknown, x .

a) $3x = 6$

b) $x + 4 = 7$

c) $x - 2 = 3$

d) $4 = 2x$

e) $9 = x + 5$

f) $2x + 1 = 5$

g) $3x - 1 = 2x + 2$

h) $2x + 2 = x + 11$

i) $\frac{5x + 10}{3} = -10$

j) $7x + 1 = -x - 15$

k) $3(x - 2) = 2(2 - x)$

l) $\frac{5x + 1}{2} = \frac{6x + 15}{2}$

2. Tim is three inches taller than Emily. Emily is 60 inches tall. Set up an equation that uses an unknown, x , for Tim's height. Then solve the equation and find Tim's height.

3. Mandy has four dollars less than Mindy. Mindy has eight dollars. How many dollars does Mandy have? Set up an equation that uses an unknown, x , for the number of dollars that Mandy has. Solve the equation for x .

Name _____ Date _____

4. Julio is twice as heavy as Julian. If Julio weighs 140 pounds, how heavy is Julian? Set up an equation that uses an unknown, x , for Julian's weight. Solve the equation for x .

5. A recipe calls for six cups of flour for a full batch of biscuits. How much flour is needed for one-third of a full batch? Set up an equation that uses an unknown, x , for the number of cups of flour needed. Solve the equation for x .

6. Three times Ardella's weight minus four pounds gives the weight of her older sister. Her older sister weighs 110 pounds. How much does Ardella weigh? Set up an equation that uses an unknown, x , for Ardella's weight. Solve the equation for x .

7. One-half of the difference between an unknown number and three is equal to one fourth of the unknown number. What is the unknown number? Set up an equation that uses an unknown, x , and then solve for x .

8. Elwood bought some apples from a street vendor. He gave the apple vendor four dollars and received 50 cents in change. The apples were 25 cents each. How many apples did he buy? Set up an equation that uses an unknown, x , for the number of apples. Then solve the equation for x .

9. The principal of a school requires that there be a certain number of playground supervisors present when young children are playing. She requires that there be one permanent supervisor plus two more supervisors for every 10 children that are on the playground. How many supervisors are needed if 25 children are on the playground? Set up an equation that uses x for the number of supervisors needed, and solve the equation for x .

Name _____ Date _____

LESSON SEVEN: GRAPHING PROBLEMS

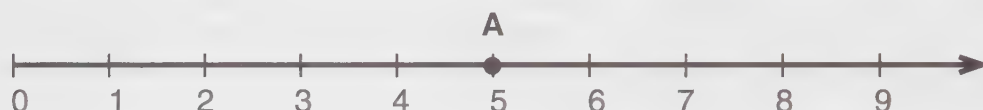
LESSON SEVEN: Exercise 1

When we are confronted with groups of numbers, we can sometimes be a bit bewildered. We would like to find the meaning of the numbers, but that meaning is not always obvious. Graphs can sometimes help us. A **graph** is a visual representation of a collection of numbers. Instead of seeing a list of numbers, we see a picture, and the picture is often easier for us to understand than the numbers.

There are many types of graphs, and they all have important uses. In this lesson, we will investigate bar graphs and rectangular coordinate graphs and some of their applications. To get started, however, we will first look at simple number lines.

Number Lines

A convenient way of showing numbers is with a number line. A **number line** is just a horizontal or vertical line on which numbers are shown. The simplest number line is the **whole number line** shown below.



It extends from 0 to a very large number (infinity.) Marks along the line, called **tick marks**, show the positions of the whole numbers. A particular whole number can be shown on the line by drawing a small dot at that number's position. The dot labeled **A** on the above number line shows the position of the number 5. This is sometimes called **plotting** the number. We can say that the number 5 has been **plotted** on the whole number line. We will **plot** many numbers in this lesson.

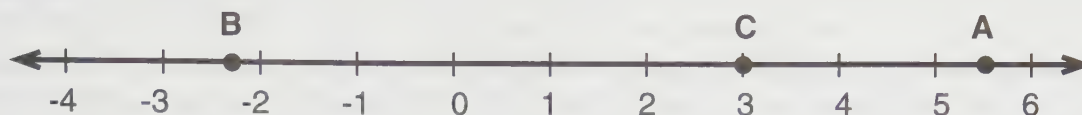
The number line below is the integer line. It extends from zero to very large positive numbers (infinity) on the right, and to very large negative numbers (minus infinity) on the left. The integer line allows both positive and negative numbers to be plotted. The integers **A** = 4, **B** = -2, and **C** = 0 are shown plotted on the integer line.



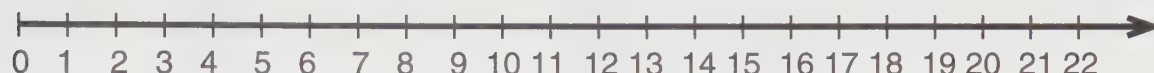
Real numbers can be located along the **real number line**. Like the integer line, it extends to infinity in both positive and negative directions. Unlike the integer line, it allows points to be plotted in between the tick marks.

Name _____ Date _____

The example below shows the real number line with the points **A** = 5.5, **B** = -2.25, and **C** = 3.0 plotted



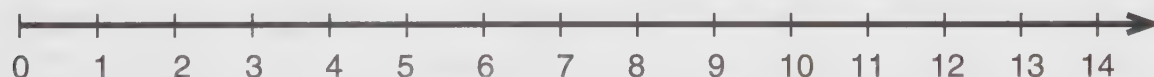
1. An elementary school had three first grade classes, Mrs. Jones' class, Mr. Martinez' class, and Miss Brown's class. There were 20 students in Mrs. Jones' class, 14 in Mr. Martinez' class, and 12 in Miss Brown's class. Plot points on the number line of whole numbers shown below that show the sizes of the three classes. Label your points **Jones**, **Martinez**, and **Brown**.



2. Mickey has six quarters, four dimes, eight nickels, and two pennies. Plot the numbers of each of his coins on the whole number line below. Label your points **Quarters**, **Dimes**, **Nickels**, and **Pennies**.

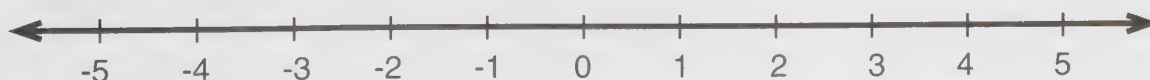


3. A pet store has seven baby rabbits, 13 kittens, and nine puppies. Plot the number of each type of baby animal on the whole number line below. Label your points **Rabbits**, **Kittens**, and **Puppies**.



4. Golf scores are sometimes recorded as the number of strokes (swings of a golf club) above or below some set number. This set number is called par and is a different number for different golf courses. Scores that are under par are recorded as negative and scores that are above par are recorded as positive integers.

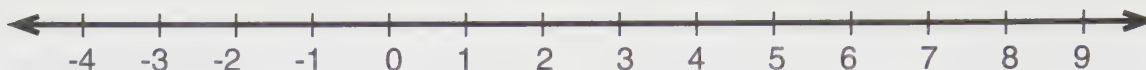
A foursome of golfers played a par 36 nine-hole golf course. Jane shot a 33 (three under par), Annette shot a 38 (two over par), Ellen shot a 40 (four over par), and Judy shot a 36 (even par). Plot each of their scores as the number above (positive) or below (negative) par on the integer line below. Write the golfer's name next to each plotted score.



Name _____ Date _____

LESSON SEVEN: Exercise 2

1. A teacher decides not to grade students with A's, B's, C's, D's, and F's. Instead, she rates each student with an integer that is between -10 and +10. On her scale, Max got a -3, Jean received a 7, Jan got a 2, and Clyde received a 0. Plot each of the students' grades on the integer line below. Write the student's name next to his or her grade.



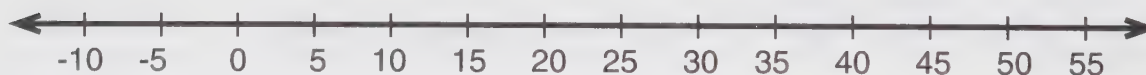
2. Edward receives an allowance of \$10 per week. His mother also lets him borrow money from her as long as he does not go more than \$10 in debt. Edward records the amount of money that he has (a positive number) or the amount that he is in debt (a negative number) at the end of each week. The list below is part of his record.

Week	Amount
April 3	\$7
April 10	-\$5
April 17	-\$3
April 24	\$1

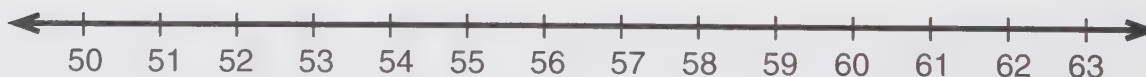
Plot Edward's money at the end of each of these four weeks on the integer line below. Write the week next to each number.



3. The average temperature in a northern city was -8.5°F in January, -0.30°F in February, 20.5°F in March, and 47.0°F in April. Plot the average temperatures on the real number line below. Write the month next to each temperature.



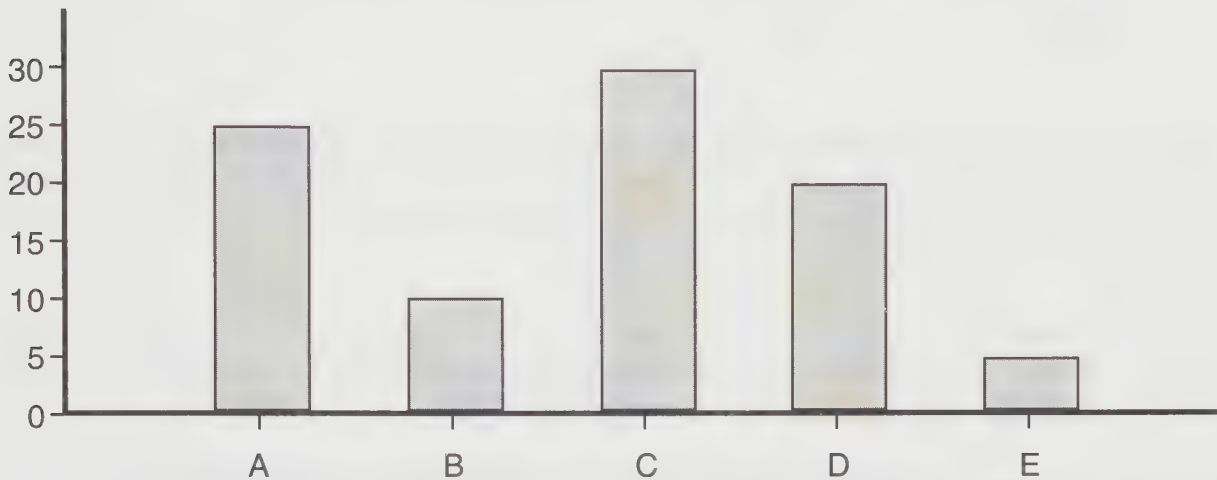
4. George was 57.5 inches tall, Ginger was 52.25 inches tall, Gavin was 61 inches tall, and Gina was 50.75 inches tall. Plot the heights of these students on the real number line below. Write each student's name next to his or her height.



Name _____ Date _____

LESSON SEVEN: Exercise 3**Bar Graphs**

Some of the numbers in the previous problems could be plotted on a **bar graph**. A bar graph is shown in the picture below. Each one of the bars in the graph shows the value of a number. The labels at the base of each bar tell what the bar represents.



We sometimes say that a graph, like the bar graph above, uses two axes. An **axis** is a line on which numbers can be plotted. The **vertical axis** on the bar graph is the one that extends up and down the page. It contains tick marks and a number scale, so that the heights of the bars can be converted to numerical values.

The **horizontal axis** extends from left to right across the page. The tick marks on it are equal distances apart. The words or numbers under the tick marks identify the bar that is drawn at that tick mark. Most of the bars are drawn upward from the horizontal axis. That is because they represent positive values. If a bar represents a negative number it is drawn downward from the horizontal axis.

Name _____ Date _____

A sample bar graph problem is shown below.

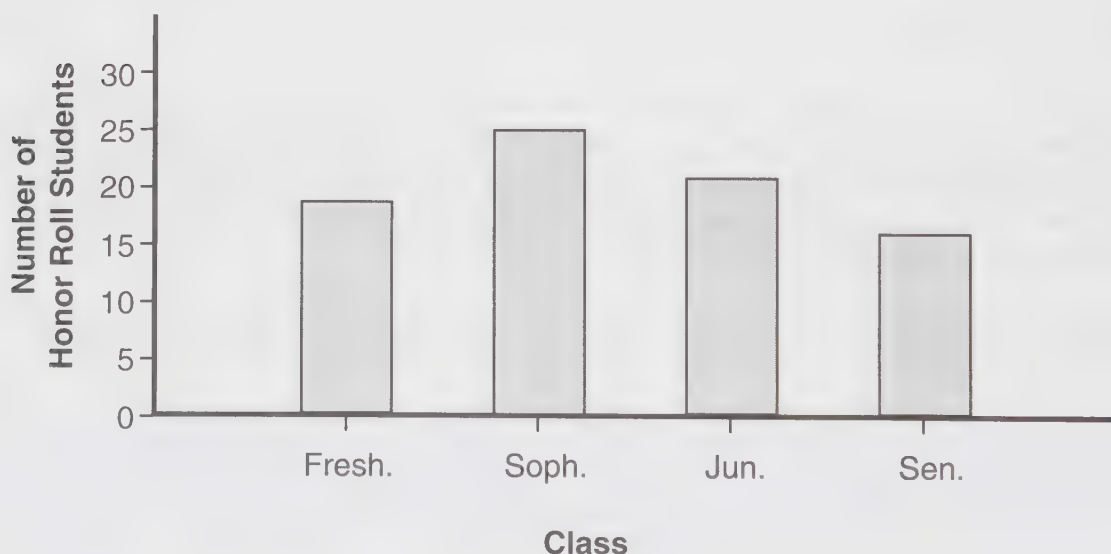
The principal at a high school added up the number of students from each class (Freshman, Sophomore, Junior, and Senior) that were on the Honor Roll for high grades. His totals are shown in the following chart.

Freshmen	18
Sophomores	25
Juniors	20
Seniors	16

He decides to show these numbers on a bar graph. First, he draws a vertical axis toward the left side of a piece of paper. He notes that the highest score is 25 and he decides to place tick marks along the vertical axis that are five units apart. He needs seven marks (if you count the one at zero.)

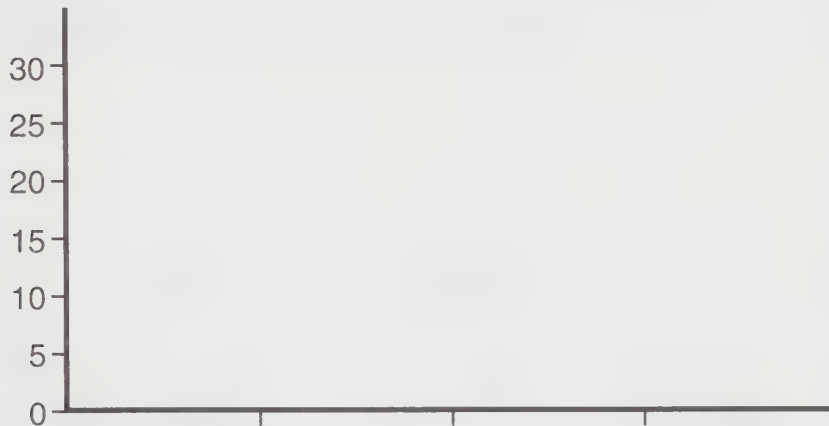
Then he draws the horizontal axis. He needs four tick marks on the horizontal axis, he also needs to write the name of the bar under each of these tick marks. Because the names in this problem are long, he uses the abbreviations, Fresh., Soph., Jun., Sen.

Finally, he draws in the bars. He makes all of the bars the same width and chooses a width that looks good (not too fat and not too skinny.) The height of each bar extends up to the number of students that it represents (as measured on the vertical axis.)



Name _____ Date _____

1. An elementary school had three first grade classes, Mrs. Jones' class, Mr. Martinez' class, and Miss Brown's class. There were 20 students in Mrs. Jones' class, 14 in Mr. Martinez' class, and 12 in Miss Brown's class. On the axes below, draw a bar graph that represents the number of students in Mrs. Jones' class, Mr. Martinez' class, and Miss Brown's class. Draw the bar that shows the number of students in Mrs. Jones' class at the left tick mark. Draw the bar that shows the number of students in Mr. Martinez' class at the middle tick mark, and draw the bar that shows the students in Miss Brown's class at the right tick mark. Place the labels, Jones, Martinez, Brown, under the correct tick mark. You will also need to label the axes of the graph.



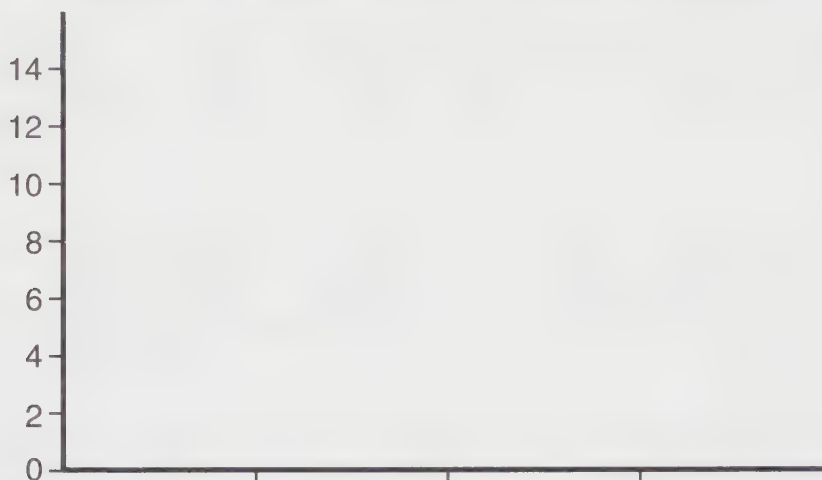
2. Mickey has six quarters, four dimes, eight nickels, and two pennies. On the axes below, draw a bar graph that shows the number of quarters, dimes, nickels, and pennies that Mickey has in his pocket. Label your bars, **Quarters**, **Dimes**, **Nickels**, and **Pennies**. Don't forget to label the axes.



Name _____ Date _____

LESSON SEVEN: Exercise 4

1. A pet store has seven baby rabbits, 13 kittens, and nine puppies. On the axes below, draw a bar graph that shows the number of each type of baby animal. Label your bars **Rabbits**, **Kittens**, and **Puppies**. Don't forget to label the axes.



2. Golf scores are sometimes recorded as the number of strokes (swings of a golf club) above or below some set number. This set number is called par and is a different number for different golf courses. Scores that are under par are recorded as negative and scores that are above par are recorded as positive integers.

A foursome of golfers played a par 36 nine-hole golf course. Jane shot a 33 (three under par), Annette shot a 38 (two over par), Ellen shot a 40 (four over par), and Judy shot a 36 (even par). On the axes below, draw a bar graph that shows the golf scores (number of strokes above or below par) for Jane, Annette, Ellen and Judy. Consider a below par score to be a negative number and an above par score to be a positive number. Write the golfer's name next to each bar score. Don't forget to label the axes.



Name _____ Date _____

LESSON SEVEN: Exercise 5

1. A teacher decides not to grade students with A's, B's, C's, D's, and F's. Instead, she rates each student with an integer that is between -10 and +10. On her scale, Max got a -3, Jean received a 7, Jan got a 2, and Clyde received a 0. Draw vertical and horizontal bar graph axes. Place the numbers -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 next to equally spaced tick marks on the vertical axis. Then place four equally spaced tick marks on the horizontal axis. Next draw a bar at the leftmost tick mark that shows Max's grade (-3.) Then, at the next tick mark, draw a bar that shows Jean's grade (7.) Then draw a bar showing Jan's grade (2.) Finally draw a bar that shows Clyde's grade (0.) Write each student's name next to his or her bar. Don't forget to label the axes.
2. The following are data showing the numbers of students participating in different sports in a small town. Create a bar graph that shows a bar for each sport. Label the tick marks on the vertical axis, 0, 10, 20, 30, 40, 50, and 60. Don't forget to label the axes.

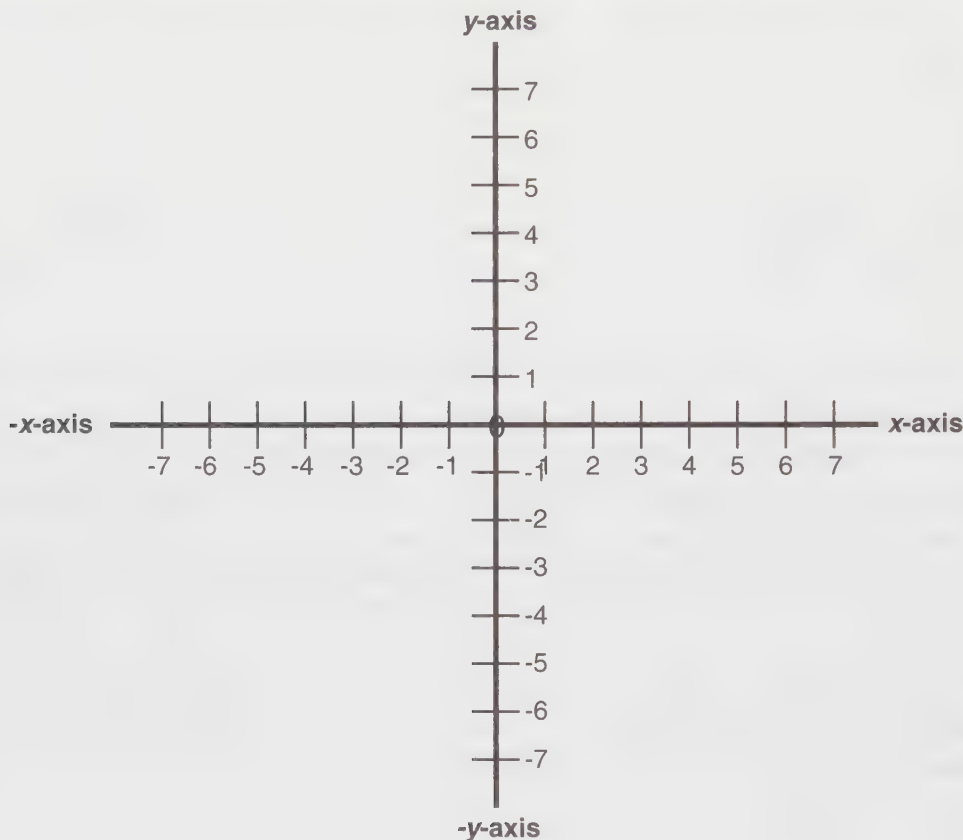
Golf	15
Tennis	10
Baseball	45
Basketball	35
Football	55
Soccer	40

Name _____ Date _____

LESSON SEVEN: Exercise 6**Coordinate Plots**

Sometimes we have numbers (often real numbers) that are in pairs. We usually write paired numbers inside of parentheses, separated by a comma. $(4, 6)$, $(-3, -7)$, and $(5, -2)$ are paired numbers. If we try to plot a pair of numbers on a number line, we find that we actually plot two numbers that do not appear to be related. We need a different way to plot these types of numbers.

Paired numbers are plotted on coordinate graphs like the one shown below.



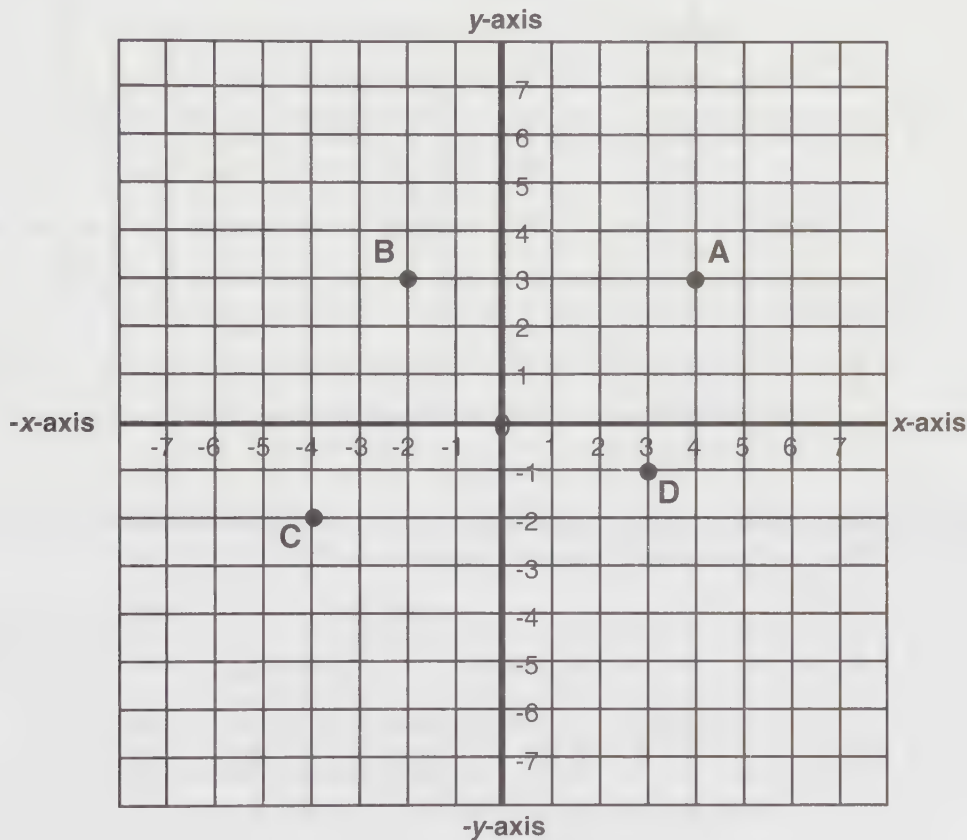
Pairs of real numbers are plotted on rectangular coordinate systems. **Rectangular coordinate systems** consist of two real number lines. One of these is called the **vertical axis** and is drawn up and down the page. The other one is called the **horizontal axis** and is drawn left and right across the page. Often, we call the horizontal axis the **x-axis**, and the vertical axis the **y-axis**.

Tick marks on both axes show the positions of numbers along the axes. They are spaced equal distances apart. The axes intersect at a point called the **origin**. The tick marks at the origin are labeled 0 in both the horizontal and vertical directions.

Name _____ Date _____

The pairs of real numbers that are plotted on rectangular coordinate systems are called **points**. The first number in the pair specifies a position along the horizontal axis (x -axis), and the second number gives a position along the vertical axis (y -axis). The actual position of the point can be found by drawing a vertical line through the number plotted on the horizontal axis and a horizontal line through the number plotted on the vertical axis. The intersection of those two lines gives the location of the point in the coordinate system.

The sample below shows four points. A (4, 3), B (-2, 3), C (-4, -2), and D (3, -1) plotted on a rectangular coordinate system.



X-Y Graphs

A type of graph called an **x - y graph** can be drawn by connecting points that are plotted on a rectangular coordinate system. Line segments are drawn from one point to another, usually in the order that the points are plotted. Sometimes the line segments are straight, but in other cases they may be curved to make a smooth line that goes through all of the points. The shape of the graph can often tell someone a lot about the points that have been plotted.

Name _____ Date _____

The x-y graph below is plotted from the points in the following table.

Points

(-2, -1)

(-1, 1)

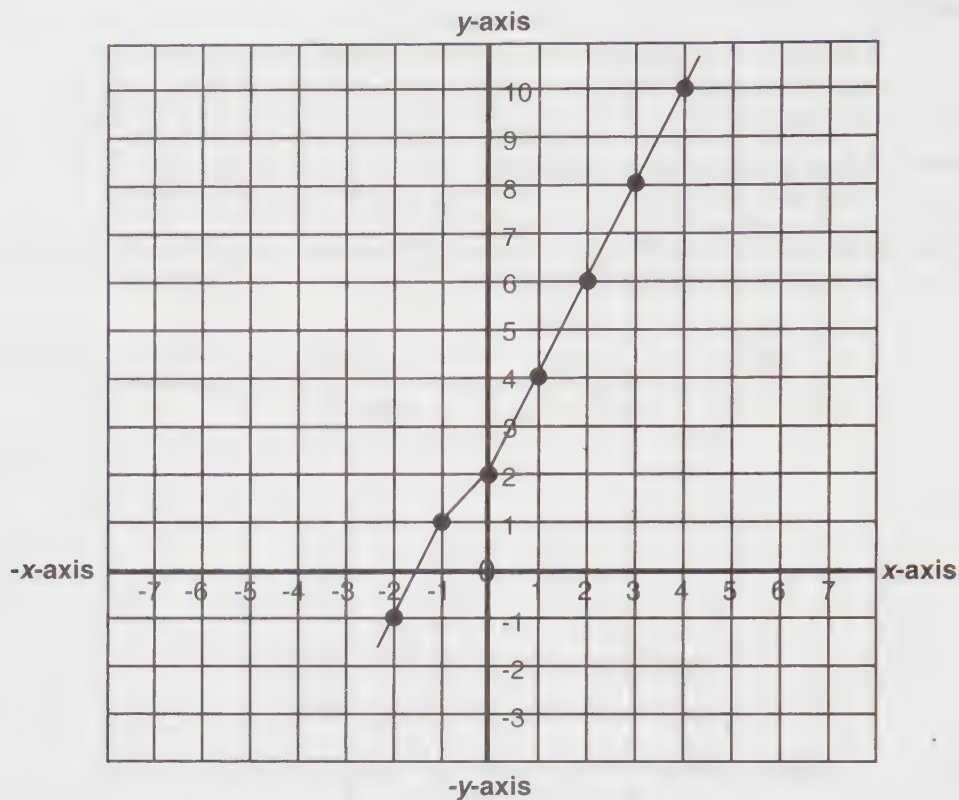
(0, 2)

(1, 4)

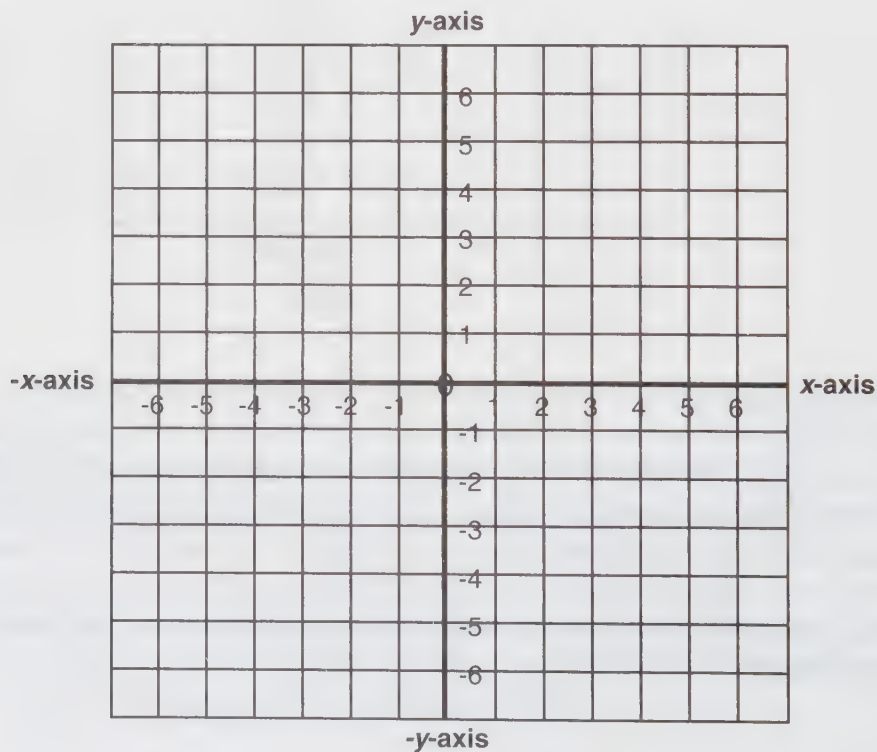
(2, 6)

(3, 8)

(4, 10)



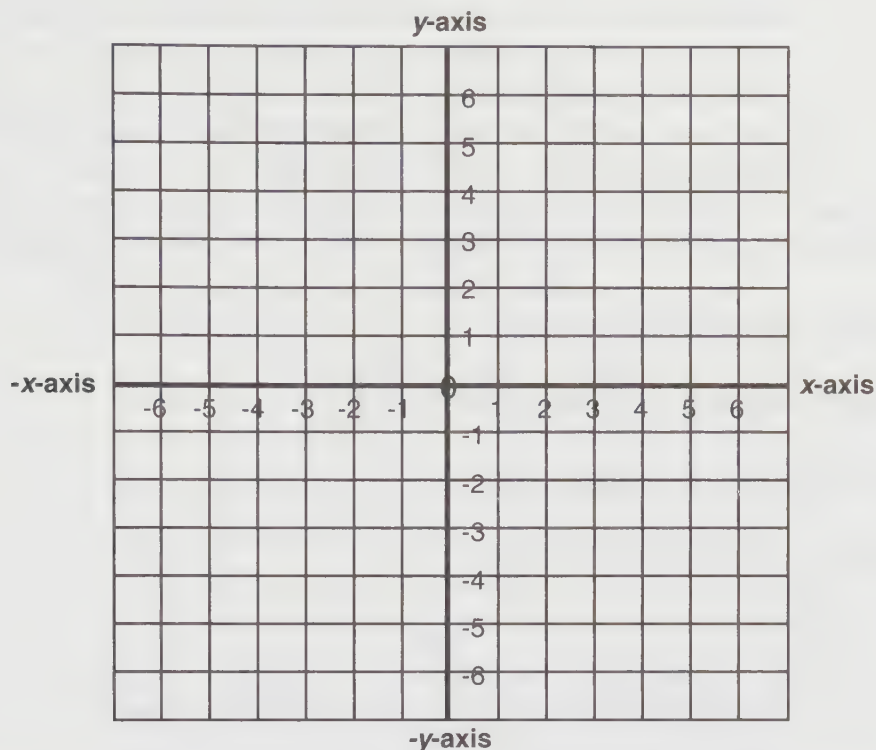
1. Plot the following points on the rectangular coordinate system below. Write the letter next to the point.

A $x = 4, y = 3$ **B** $x = -2, y = 5$ **C** $x = -4, y = -4$ **D** $x = 5, y = -1$ 

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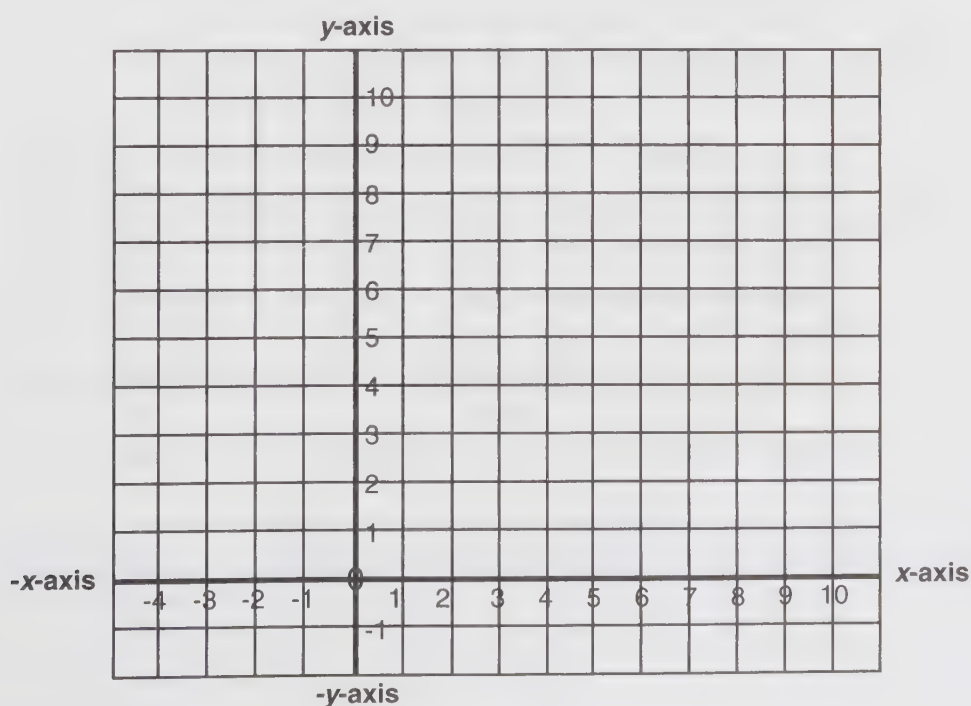
2. Plot the following points on the rectangular coordinate system below. Write the letter next to the point

A (6, 2) B (-5, 3) C (-3, -3) D (2, -4)



3. Plot the following points on the rectangular coordinate system below. Then connect the points in the order that they are listed. What figure do you see?

(1, 0) (10, 0) (10, 4) (6, 8) (3, 6) (3, 7) (2, 7) (2, 5) (1, 4) (1, 0)



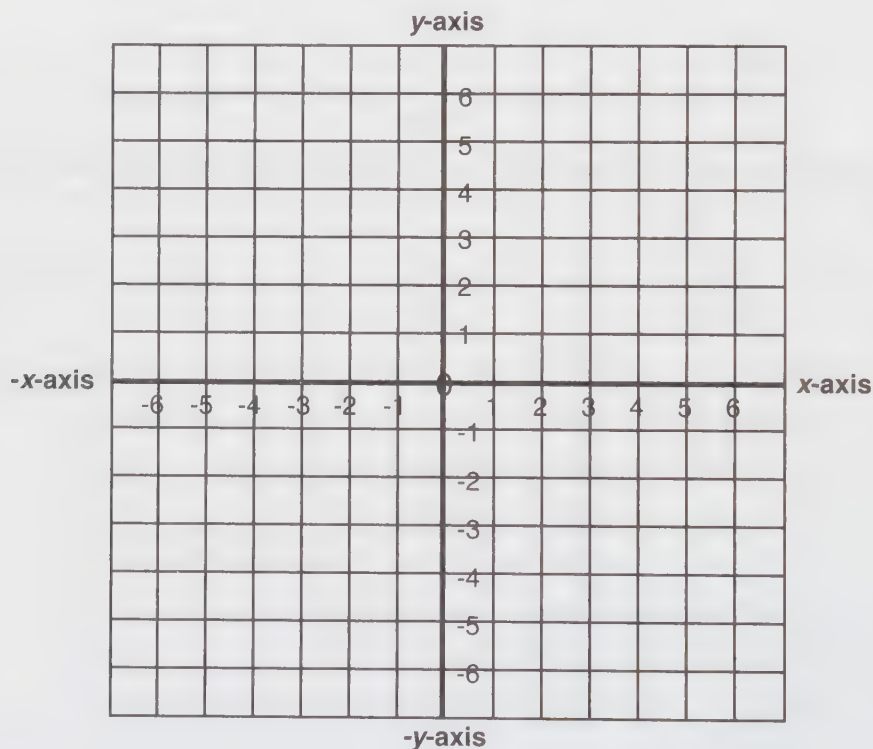
Name _____ Date _____

LESSON SEVEN: Exercise 7

1. Sean lives in a big city where the houses are arranged in square blocks. He sometimes remembers directions to other places by remembering how many blocks east, west, north, or south he must travel from his house to get to the other place.

Use the rectangular coordinates below to represent Sean's house and the square blocks around it. Sean's house is at 0. Each tick mark on the graph represents one block of distance. Let east be the positive x direction and west be in the negative x direction. Let north be the positive y direction and south be in the negative y direction.

- a) **A** 2 blocks east and 3 blocks north
- b) **B** 4 blocks west and 5 blocks north
- c) **C** 5 blocks west and 2 blocks south
- d) **D** 6 blocks east and 4 blocks south



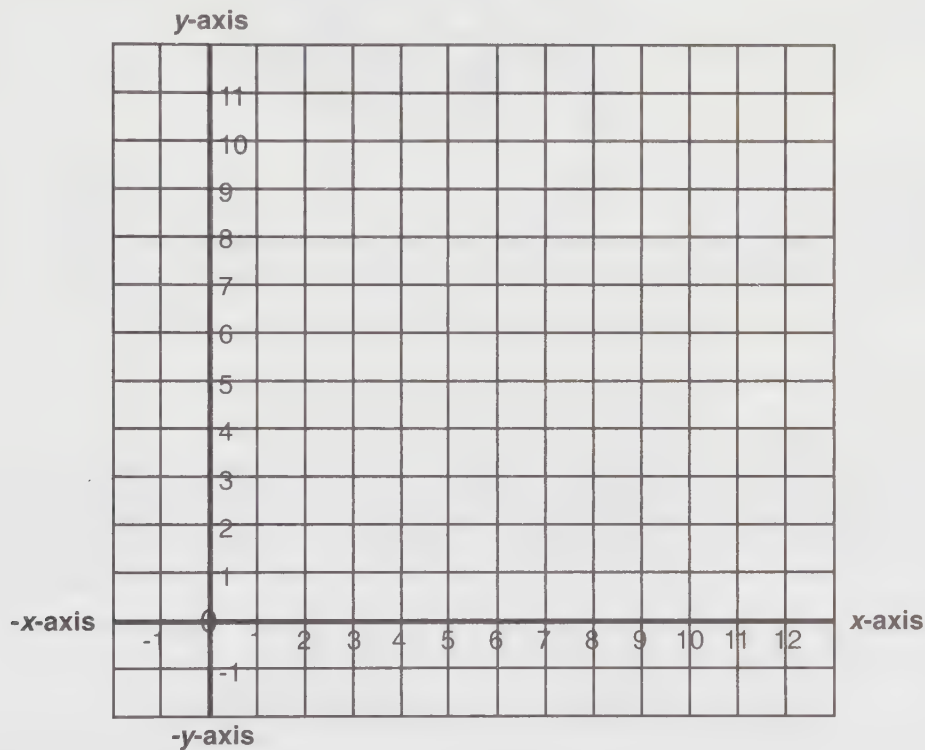
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2. TyAnn is investigating the slope of the mountainside in back of her home in Colorado. She has a very sensitive measuring instrument that tells her how far she is from her back yard in the horizontal (x) direction, and how far she is above her back yard in the vertical (y) direction. **Both distances are in hundreds of feet.**

- a) TyAnn walks away from her house and climbs up the mountainside. She gets the following pairs of measurements.

(0, 0) is her starting point.

(1, 1.5) (2, 2) (3, 3) (4, 3.5) (5, 3.5) (6, 4) (7, 4.5) (8, 5) (9, 5.5) (10, 6)
(11, 8) (12, 8.5)



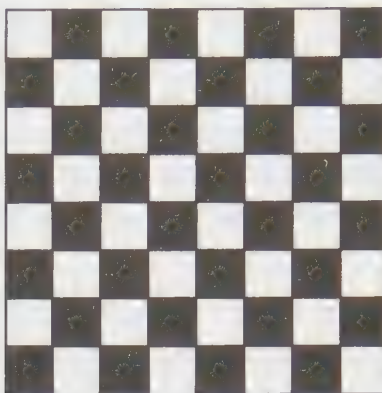
- b) Plot the points on the rectangular coordinates above.
- c) Draw lines between the points in the order in which you plotted them.
- d) Between which two points is the mountainside the steepest? _____
- e) Between which two points is the mountainside the least steep? _____
- f) How far away, in the horizontal direction, does TyAnn travel from her backyard?

- g) How far up, in the vertical direction, does TyAnn travel above her backyard?

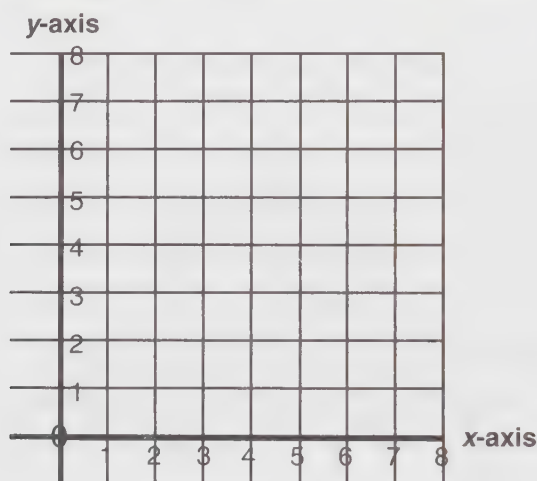
Name _____ Date _____

LESSON SEVEN: Exercise 8

Chess is a strategy game that has been played since the middle ages. It is played on a chessboard, like the one shown below, that is divided into squares.



The chessboard is eight squares by eight squares in size. It is possible to represent the positions of the squares on a rectangular coordinate graph like the one below. Each tick mark on the graph represents one of the squares on the chessboard.

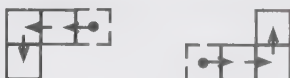


Chess is played by moving pieces, called chessmen, from square to square, on the board. Different types of chessmen have different moves that they are allowed to make. One particular type of chessman, called the knight, can make the following types of moves.

- 1) Two squares up or down on the board, followed by one square to the left or right.



- 2) Two squares to the left or right on the board, followed by one square up or down.



Name _____ Date _____

1. A knight is on the (0, 0) square. He moves two squares up and one square to the right (one knight move.) What square is he now on?

2. A knight is on the (0, 0) square. He moves two squares to the right and one square up (one knight move.) What square is he now on?

3. A knight is on the (4, 5) square. He moves two squares down and one square to the left. He then moves two squares to the right and one square up. (He has made two knight moves.) What square is he now on?

4. A knight is on the (7, 7) square. He moves two squares down and one square to the left. He then moves two squares to the left and one square down. (He has made two knight moves.) What square is he now on?

5. A knight is on the (4, 4) square. He moves two squares up and one square to the left. He then moves two squares to the right and one square down. Finally, he moves two squares down and one square to the left. (He has made three knight moves.) What square is he now on?

6. A knight is on the (5, 5) square. He moves two squares down and one square to the left. He then moves two squares to the left and one square down. Finally, he moves two squares up and one square to the right. (He has made three knight moves.) What square is he now on?

7. A knight is on the (5, 6) square. What is the smallest number of knight moves that he must make to get to the (2, 3) square?

8. A knight is on the (4, 4) square. What is the smallest number of knight moves that he must make to get to the (5, 3) square?

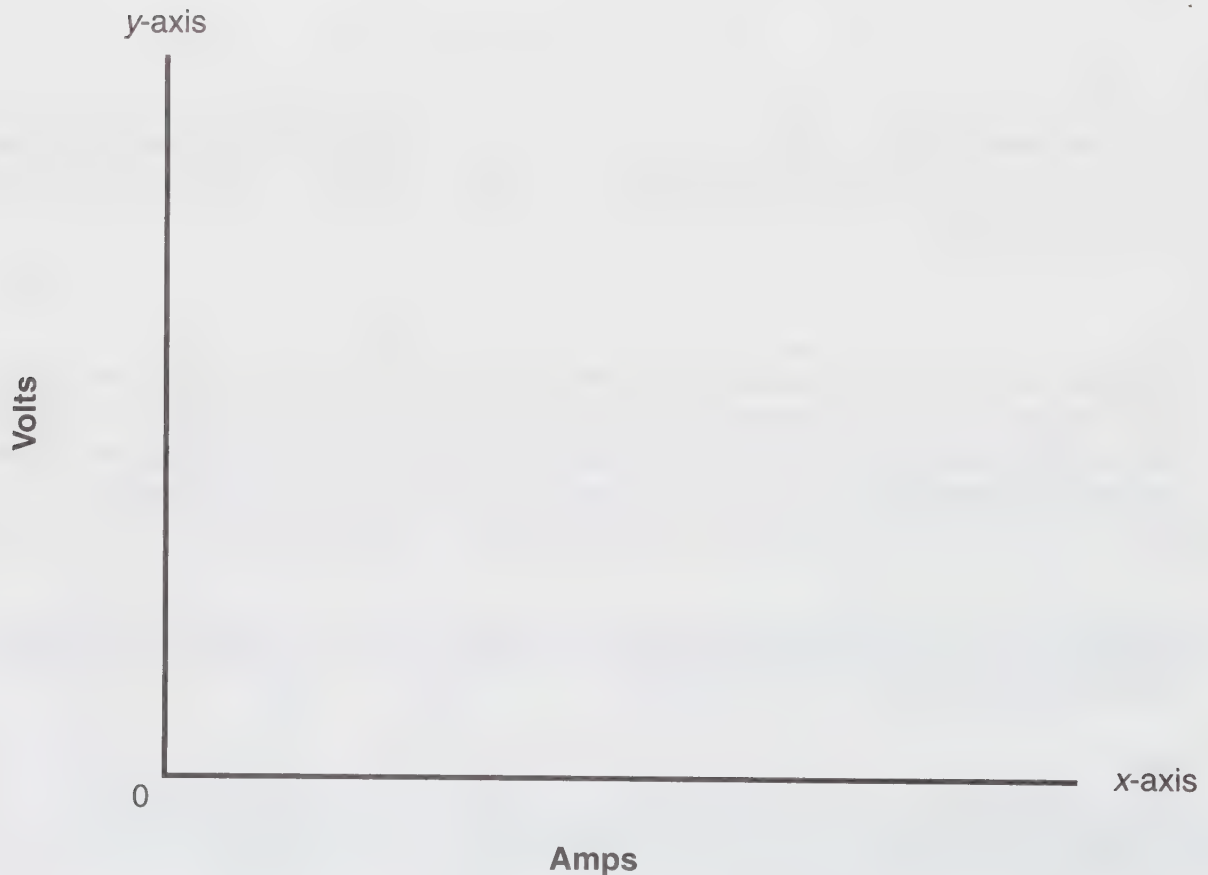
Name _____ Date _____

LESSON SEVEN: Exercise 9

1. Geraldine is doing an experiment in electricity. She has several batteries that are rated at different voltages and a meter that registers electric current in amps. She connects each of the batteries to a small electric heater and measures the amount of electric current that the heater draws. Her data are listed in the columns below.

On the coordinate axes that are provided, plot the points that Geraldine obtained from her experiment. Plot the voltage on the vertical axis and the current on the horizontal axis. Place your own tick marks along the x and y axes.

Battery	Voltage	Current
A	5 Volts	2.5 Amps
B	8 Volts	4.0 Amps
C	10 Volts	5.0 Amps
D	12 Volts	6.0 Amps
E	15 Volts	7.5 Amps
F	18 Volts	9.0 Amps



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LESSON EIGHT: TWO-VARIABLE EQUATION PROBLEMS

In Lesson Six, you found values that were specified by formulas and solved problems that involved an unknown. In other problems there may be two unknowns in an equation. A particular type of equation with two unknowns is called a linear equation with two variables. In this lesson, we will investigate this type of equation and some simple examples of other ones.

Linear Equations With Two Variables

A linear equation with two variables contains two unknowns. We will call these unknowns **variables** because their values can vary. In the equation, the variables are never squared, cubed, or taken to a higher power. In addition, there are no products of the variables in the equation.

The simplest equation with two variables is shown below. (We will use x and y for the two variables, although other symbols could be used.)

$$y = x$$

If we specify a value for x , we can quickly calculate a value for y . For example, if x is 4, y is also 4. If the value of one is given, the value for the other can be calculated. The equation can also be manipulated into other forms. For example, if x is subtracted from both sides, the equation can be written as the difference shown below.

$$y - x = 0$$

Most equations with two variables are more complicated than the above one. Often, one or both of the variables is/are multiplied by a constant number.

$$3x = 4y$$

$$3x - 4y = 0$$

One or both of the constants can be negative.

$$-3x = -4y$$

$$-3x + 4y = 0$$

A third constant, which is not multiplied times a variable, can also be present.

$$2y + 3x = 4$$

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Solving for a Variable

Linear equations with two variables can be solved for either of the two variables. To solve for a variable means to manipulate the equation so that the variable you are solving for is by itself on one side of an equal sign. An expression involving the other variable and some numbers is on the other side of the equal sign.

As an example, we will solve the linear equation below for y .

$$2y + 3x = 12$$

$$-3x + 2y + 3x = -3x + 12 \quad (\text{subtract } 3x \text{ from both sides})$$

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{12}{2} \quad (\text{divide both sides by } 2)$$

$$y = -1.5x + 6 \quad (\text{simplify})$$

Graphs of Linear Equations With Two Variables

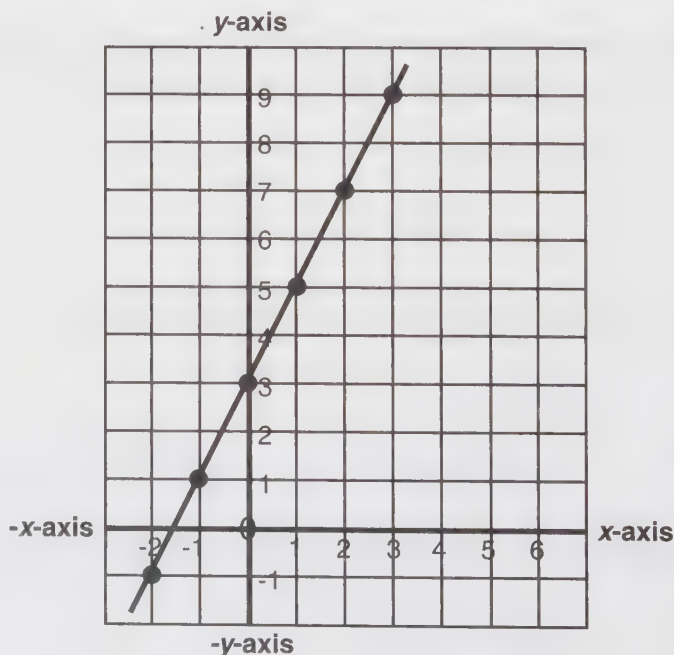
The equations that we have been looking at are called **linear equations**. The reason that they are called linear becomes obvious when we plot the equations on rectangular coordinate systems. The graphs of linear equations are straight lines.

To graph an equation, it is first necessary to get some coordinate values. In some cases, you may not have the equation, but you will have a table of values instead. You simply plot the graph, as we did in Lesson Seven. If the graph turns out to be a straight line, then you have plotted a graph of a linear equation. The following example shows a linear equation plot from a table of values.

Table of Values

x	y
-2	-1
-1	1
0	3
1	5
2	7
3	9

The graph of this equation is a straight line. Therefore, the equation is a linear equation.



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If you have the equation, and you know that the equation is linear, then you also know that the graph of the equation will be a straight line. **You only need to plot two points on your graph to completely define your straight line.** You only need to do the following:

- 1) Solve your linear equation for y .
- 2) Specify two x values.
- 3) From your equation, get two y values from your two x values. You now have two points to plot.
- 4) Plot your two points on your graph.
- 5) Use a ruler or other straight edge to draw a straight line that passes through both of your points.

The example below shows how to quickly obtain a graph of a linear equation in two variables.

Equation: $3y - 12x = 9$

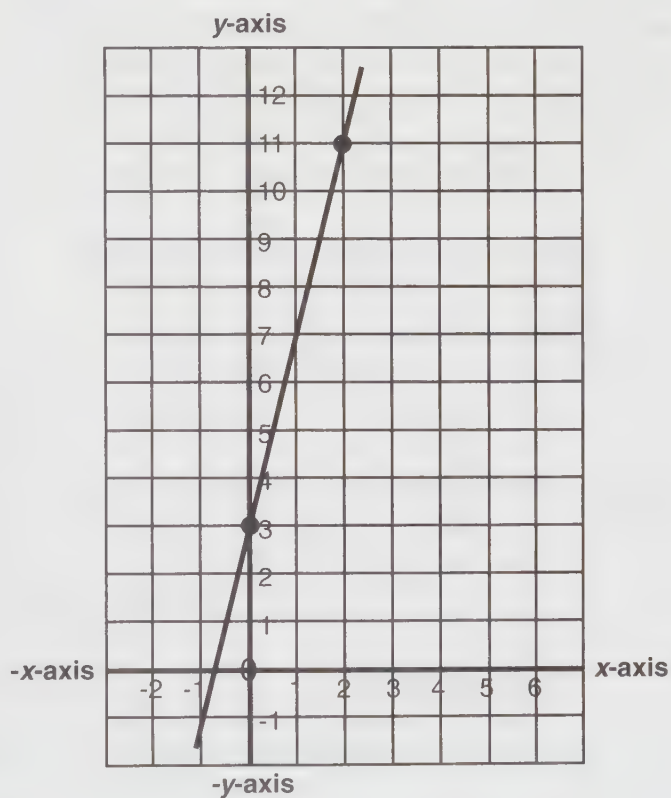
Solve for y : $y = 4x + 3$

When $x = 0$; $y = 4(0) + 3$
 $y = 3$

When $x = 2$, $y = 4(2) + 3$
 $y = 11$

Plot the Points $(0, 3)$ and $(2, 11)$.

Draw a straight line through the two points.



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In some cases, you may have an equation that is not linear, or that you are not certain is linear. The following steps will allow you to plot such an equation.

- 1) Solve your equation for y .
- 2) Make a table with two columns, one for x and one for y .
- 3) In your x column, write down the x values that you want in your coordinates.
- 4) From your solved equation in step (1), determine a y value for each of your x values. Write your y and x values in your table.
- 5) Plot the values from your table on your graph and connect your points with line segments.
- 6) Look carefully at your graph. If it is a straight line, then your equation is linear. If it is not a straight line, then your equation is of some other type.

The following example shows how to plot an equation. Although the equation is linear, we will pretend that we did not know that before we plotted the graph.

Equation: $y + 2x = 5$

Solve for y : $y = -2x + 5$

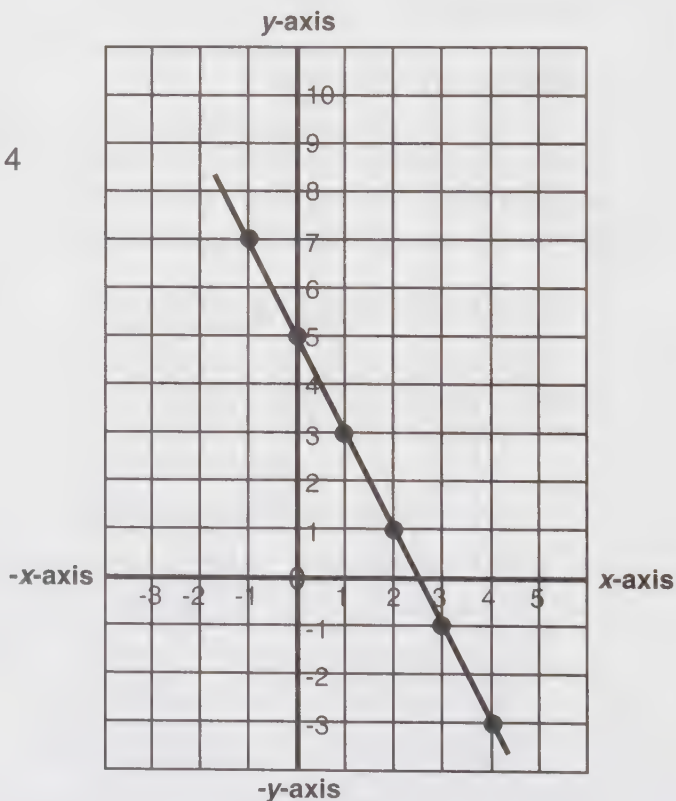
Determine x values: -1, 0, 1, 2, 3, 4

Table of Values

x	y
-1	7
0	5
1	3
2	1
3	-1
4	-3

Plot the equation.

Look at the shape of the graph.

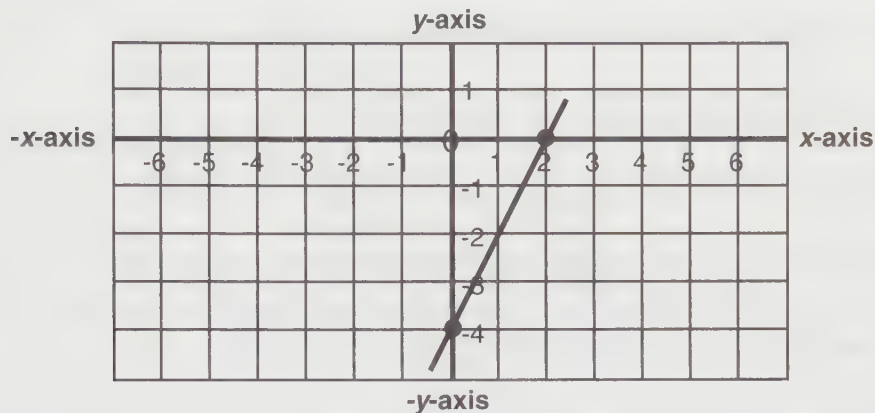


The graph of this equation turns out to be a straight line. Therefore, the equation is linear.

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Slopes and Intercepts of Linear Graphs

Linear equations in two variables produce straight-line graphs. Sometimes we are interested in the point where the line crosses the horizontal axis (x -axis) or the point where the line crosses the vertical axis (y -axis.) The point where a line crosses the x -axis is called the horizontal intercept or x -intercept, and the point where a line crosses the y -axis is called the vertical intercept or y -intercept. The picture below shows a graph of the equation $y = 2x - 4$. It has a horizontal intercept at $x = 2$ and a vertical intercept at $y = -4$.



The x -intercept and y -intercept values can be found directly from the linear equation. To find the x -intercept value from the equation, set y equal to 0 and solve for the value of x .

Example: Find the x -intercept of $2y + 3x = 6$

$$2(0) + 3x = 6$$

$$3x = 6$$

$$x = 2$$

To find the y -intercept value from the equation, set x equal to 0 and solve for the value of y .

Example: Find the y -intercept of $2y + 3x = 6$

$$2y + 3(0) = 6$$

$$2y = 6$$

$$y = 3$$

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We may also be interested in how steeply the line slants upward or downward to the right. A number, called the slope of the line, tells us how steeply it slants and whether it slants upward or downward to the right.

The slope can be found from the graph of a linear equation. The steps below show you how to obtain it.

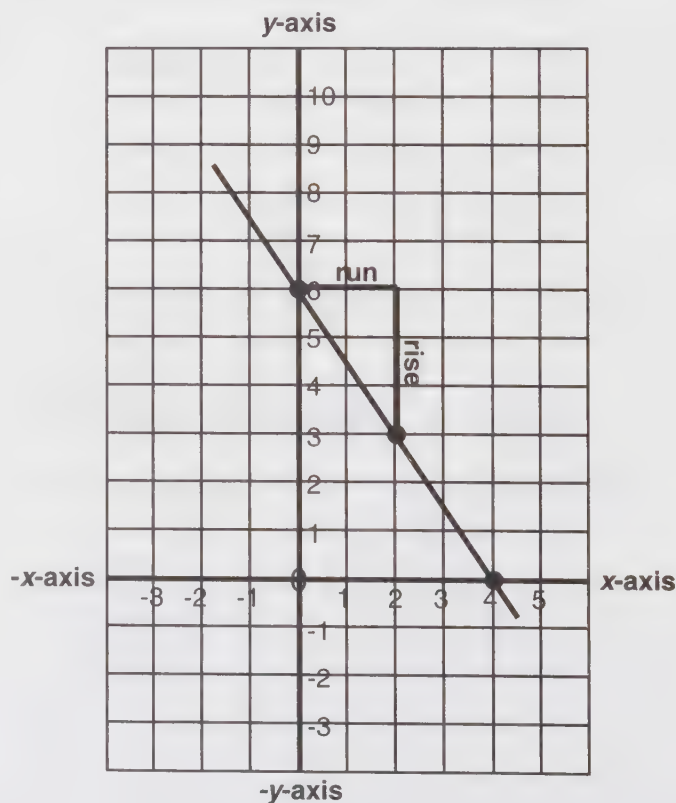
- 1) Mark two points on the graph of the linear equation.
- 2) Draw a horizontal line across the graph through one of the points.
- 3) Draw vertical line on the graph through the other point.
- 4) Notice the point where the horizontal and vertical lines intersect.
- 5) Measure distance from the intersection point from (4) vertically to the line (in graph units.) Call this distance the **rise**. The distance is positive if it slants upward to the right, and negative if the line slants downward to the right.
- 6) Measure the distance from the intersection point from (4) horizontally to the line (in graph units.) Call this distance the **run**. Always treat this distance as a positive one.
- 7) Compute the slope from the following equation.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

If the slope value turns out to be positive, the graph of the equation slants upward to the right. If the slope turns out to be negative, the graph of the equation slants downward to the right.

The size of the slope indicates how steep the line slants. A large number for the slope describes a line that slants very steeply upward or downward. A small number describes a line with very little slant. The slope of a horizontal line is zero. The slope of a vertical line is infinite.

The example at right illustrates this calculation. The equation is $2y + 3x = 12$. The slope of the line is -1.5 .



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The two points selected for the calculation are (0, 6) and (2, 3). The rise (-3) and the run (2) are indicated on the graph. The slope can be calculated as follows.

$$\text{slope} = -\frac{3}{2} = -1.5$$

The slope and y-intercept can also be determined directly from the linear equation. If the equation is solved for y , the coefficient of x on the right-hand side is the slope and the constant term (number) on the right-hand side is the y-intercept.

Example: Find the slope and y-intercept of the linear equation,
 $4y + 3x = 24$.

Solve for y .

$$y = \frac{3}{4}x + 6$$

The slope is the coefficient of x and the y-intercept is the constant.

$$\text{slope} = \frac{3}{4}$$

$$\text{y-intercept} = 6$$

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LESSON EIGHT: Exercise 11. Solve the following linear equations for x .

a) $7y + x = 11$

b) $8y + 2x = 4$

c) $6y - 3x = 36$

d) $3y + 4x = 16$

2. Solve the following linear equations for y .

a) $5y = 15x$

b) $y + 9x = 4$

c) $6y - 3x = 36$

d) $3y + 9x = 21$

3. Find the x -intercept and the y -intercept of the following linear equations.

a) $y + x = 10$

b) $5y + 4x = 40$

c) $4y - 3x = 36$

d) $7x - 4y = 70$

4. Find the slope and the y -intercept of the following equations.

a) $y - 5x = 11$

b) $3y - 12x = 24$

b) $5y + 4x = 20$

d) $4y + 3x = 0$

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LESSON EIGHT: Exercise 2

1. In the following problems, the slope and the y -intercept of a linear equation are given. Write the linear equation as $y = \text{slope } x + y\text{-intercept}$.

a) Slope = 3, y -intercept = -5

b) Slope = -2, y -intercept = 7

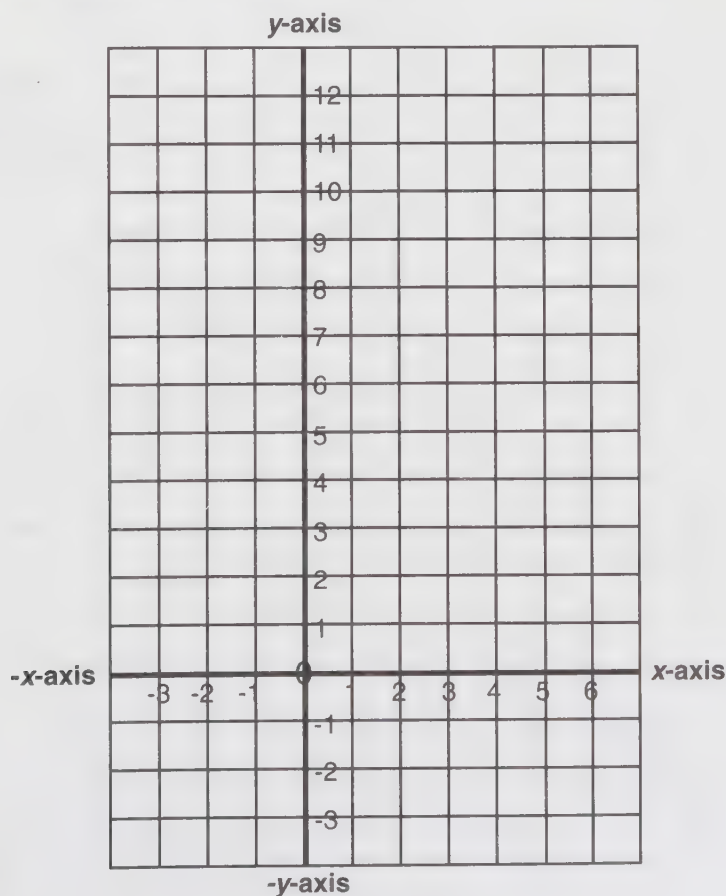
c) Slope = $\frac{1}{2}$, y -intercept = 2

d) Slope = -4, y -intercept = -9

e) Slope = $-\frac{1}{4}$, y -intercept = -1

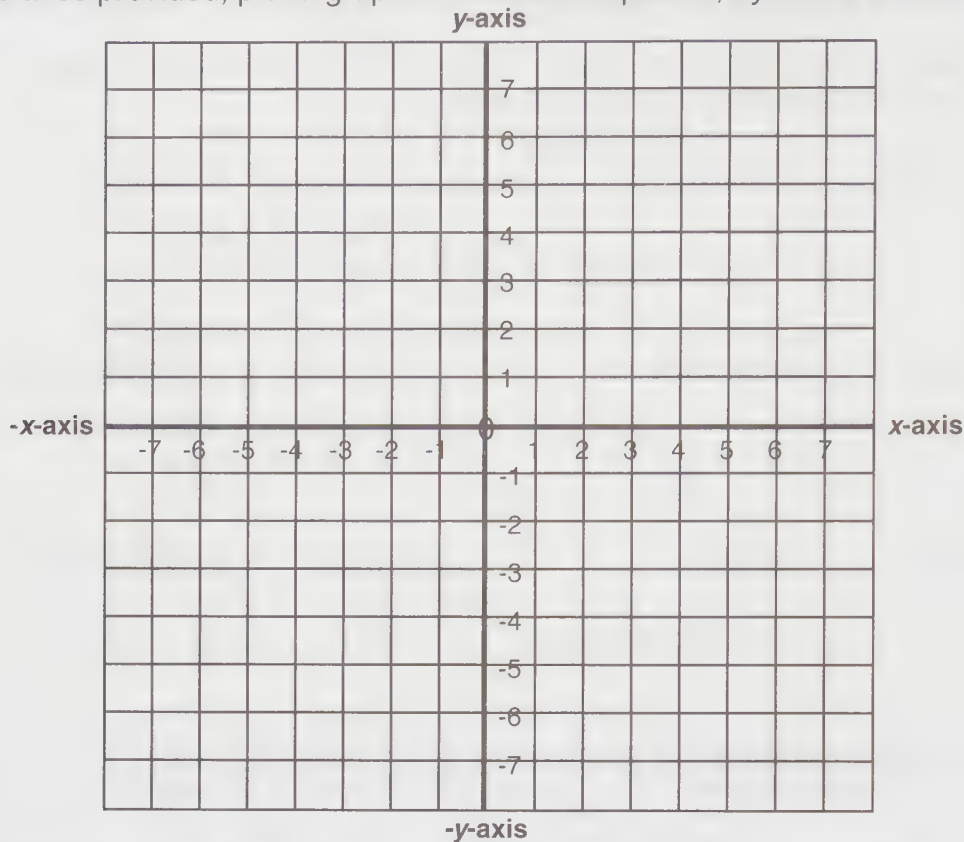
f) Slope = 0, y -intercept = 4

2. On the axes provided, plot a graph of the linear equation, $y = 2x + 3$.

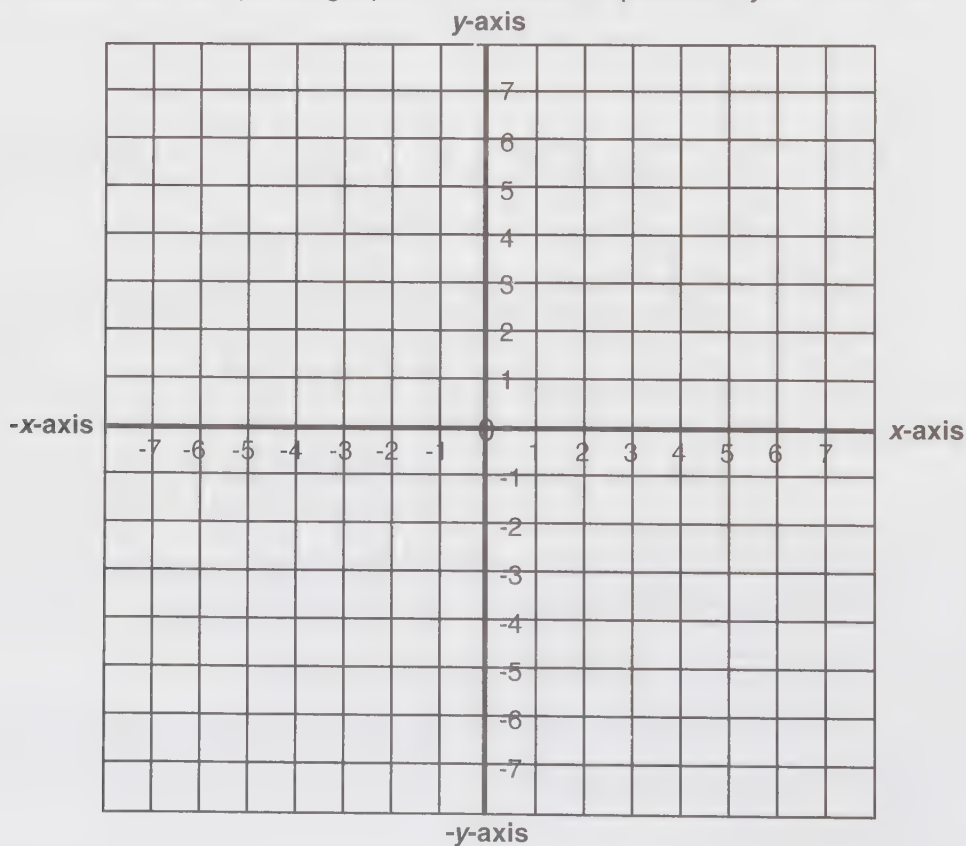


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3. On the axes provided, plot a graph of the linear equation, $2y - 5x + 4 = 0$.



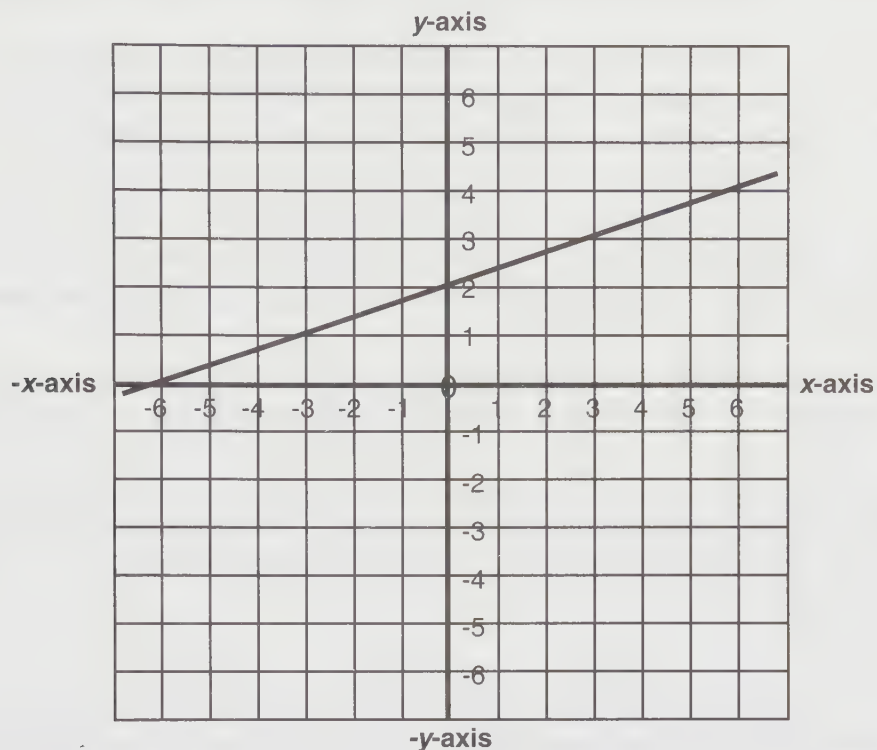
4. On the axes provided, plot a graph of the linear equation, $3y + 12x = 18$



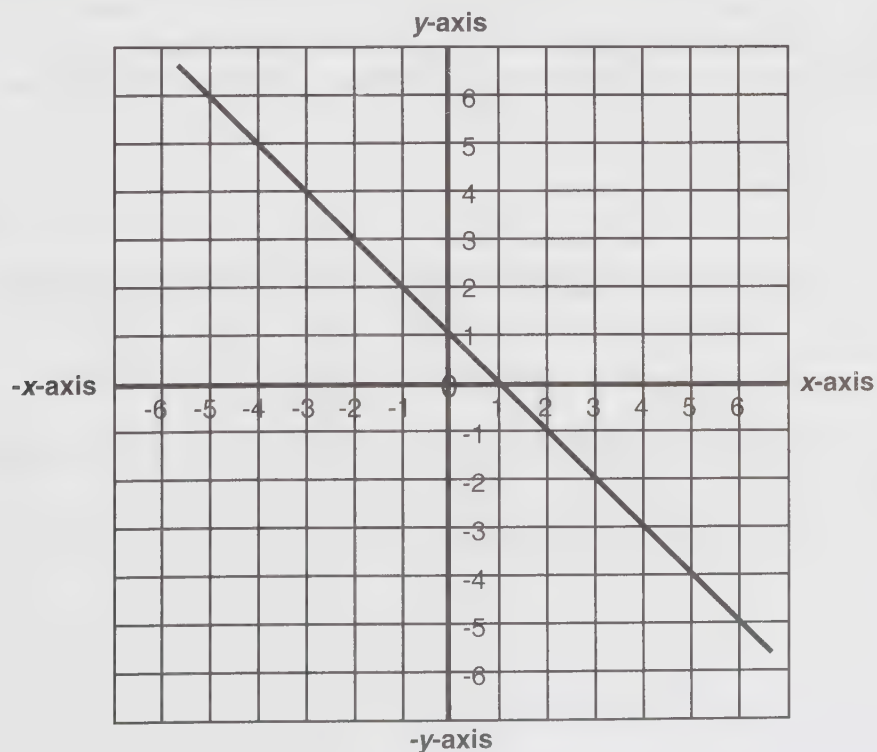
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LESSON EIGHT: Exercise 3

1. The graph below shows a plot of a linear equation. From the graph, find the slope and the y-intercept. Use these values to write the equation of the line



2. The graph below shows a plot of a linear equation. From the graph, find the slope and the y-intercept. Use the values to write the equation of the line.



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LESSON EIGHT: Exercise 4

1. Jack and Peg want to rent a limousine for prom night. The limousine company charges a flat fee of \$20 plus \$0.50 per mile.
 - a) Write a linear equation that relates the amount that Jack and Peg must pay for the limousine to the number of miles driven. Let y be the amount that they have to pay and let x be the number of miles that they ride in the limousine.
 - b) What is the slope of this equation? What does the slope represent in the problem?
 - c) What is the y -intercept in this equation? What does the y -intercept represent in the problem?

2. The ninth grade class at Raccoon Ridge Junior High School wants to have a class picnic at the end of the school year. They can rent a recreation area with a carnival and rides for \$300 plus \$5 per student.
 - a) Write a linear equation that relates the amount that the class must pay for the recreation area to the number of students who attend the picnic. Let y be the amount that must be paid and let x be the number of students who attend the picnic.
 - b) What is the slope of this equation? What does the slope represent in the problem?
 - c) What is the y -intercept in this equation? What does the y -intercept represent in the problem?

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LESSON EIGHT: Exercise 5

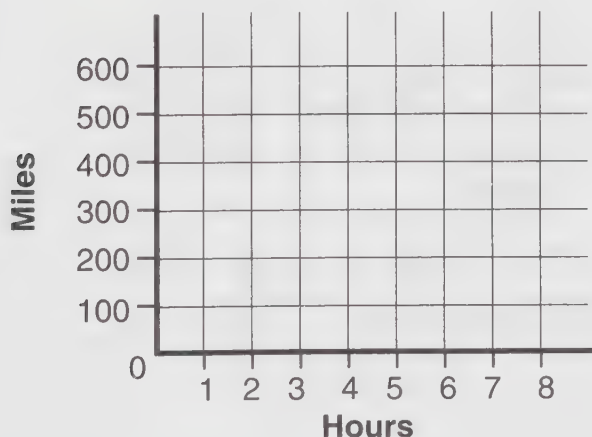
1. In physics and engineering, the distance of a moving body from some reference point (sometimes called a zero point) is given by a formula that is a linear equation.

$$d = vt + d_0$$

The **d** in this equation is the distance from some reference point, **v** is the speed at which the body is moving, and **t** is the time that the body has traveled. The **d₀** is the distance of the body from the reference point when it begins to move. In problems that use this equation, the **v** and **d₀** values are usually known. The problem is expressed as a linear equation with **d** and **t** taking the place of **y** and **x**.

Suppose that a car begins driving on a highway that runs straight west from Cheyenne, Wyoming. Also suppose that it starts at a spot that is 30 miles west of Cheyenne, and it travels at 70 miles per hour.

- a) Write a linear equation that relates distance from Cheyenne to the number of hours traveled. Let **d** be the distance traveled, and let **t** be the number of hours traveled.
- b) In the space below, plot a graph of your equation.



- c) Identify the vertical intercept and find the slope of the graph.

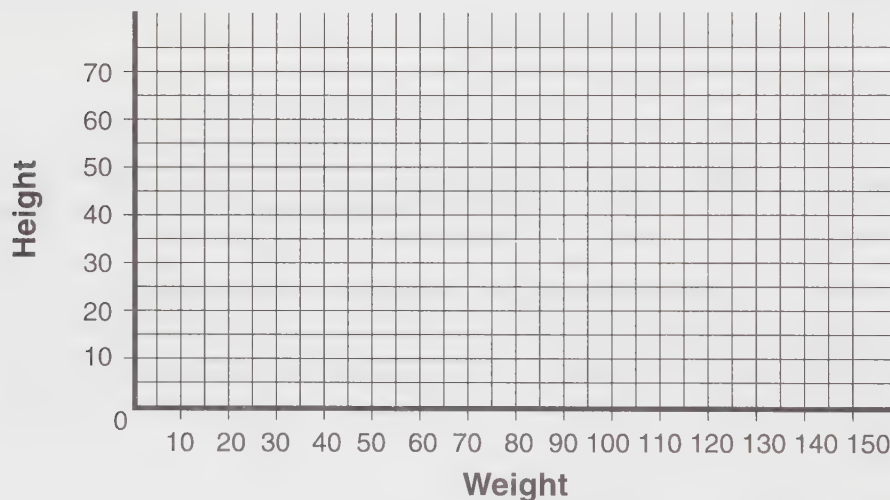
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LESSON EIGHT: Exercise 6

1. Lucinda thought that there might be a linear relationship between the weights of girls on her volleyball team and their heights. To find out, she obtained the weights (in pounds) and heights (in inches) of the following girls.

	Height H	Weight W
Mandy	60 inches	115 pounds
Sonja	62 inches	119 pounds
Lucy	65 inches	125 pounds
Rosa	68 inches	131 pounds
Jill	70 inches	135 pounds
Sally	74 inches	143 pounds

- a) On the axes below, draw a graph of the height (on the vertical axis) versus weight (on the horizontal axis).



- b) Is the equation described by the graph linear? _____
- c) What is the slope? _____
- d) What is the vertical intercept? _____
- e) Write the linear equation. Use the variables **H** for height and **W** for weight.
- _____

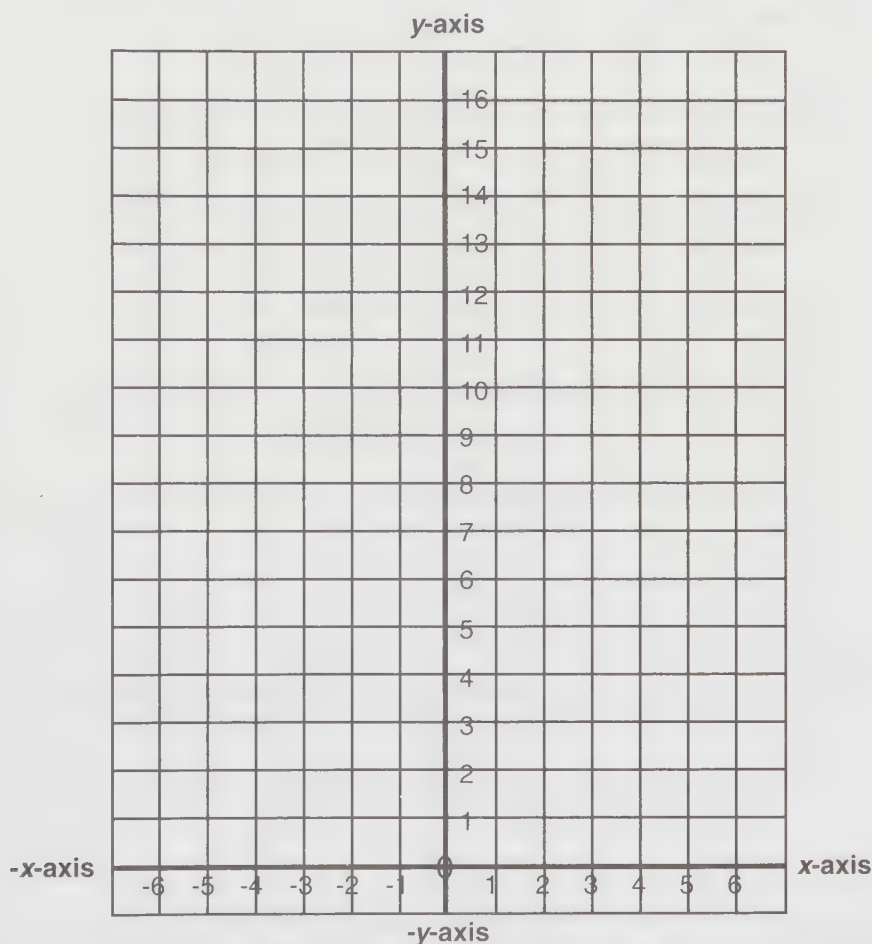
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LESSON EIGHT: Exercise 7

Not all equations are linear ones. When you study algebra, you will encounter some very interesting and very important non-linear equations. Three of these equations, in their simplest forms, are introduced in the following problems.

1. A common non-linear equation is one that shows a **quadratic** relationship between y and x . The right-hand side of such an equation always involves x squared. The simplest quadratic equation is $y = x^2$. A data table for this equation for $x = -4$ to $x = 4$ is shown below. On the axes provided, plot a graph of this equation.

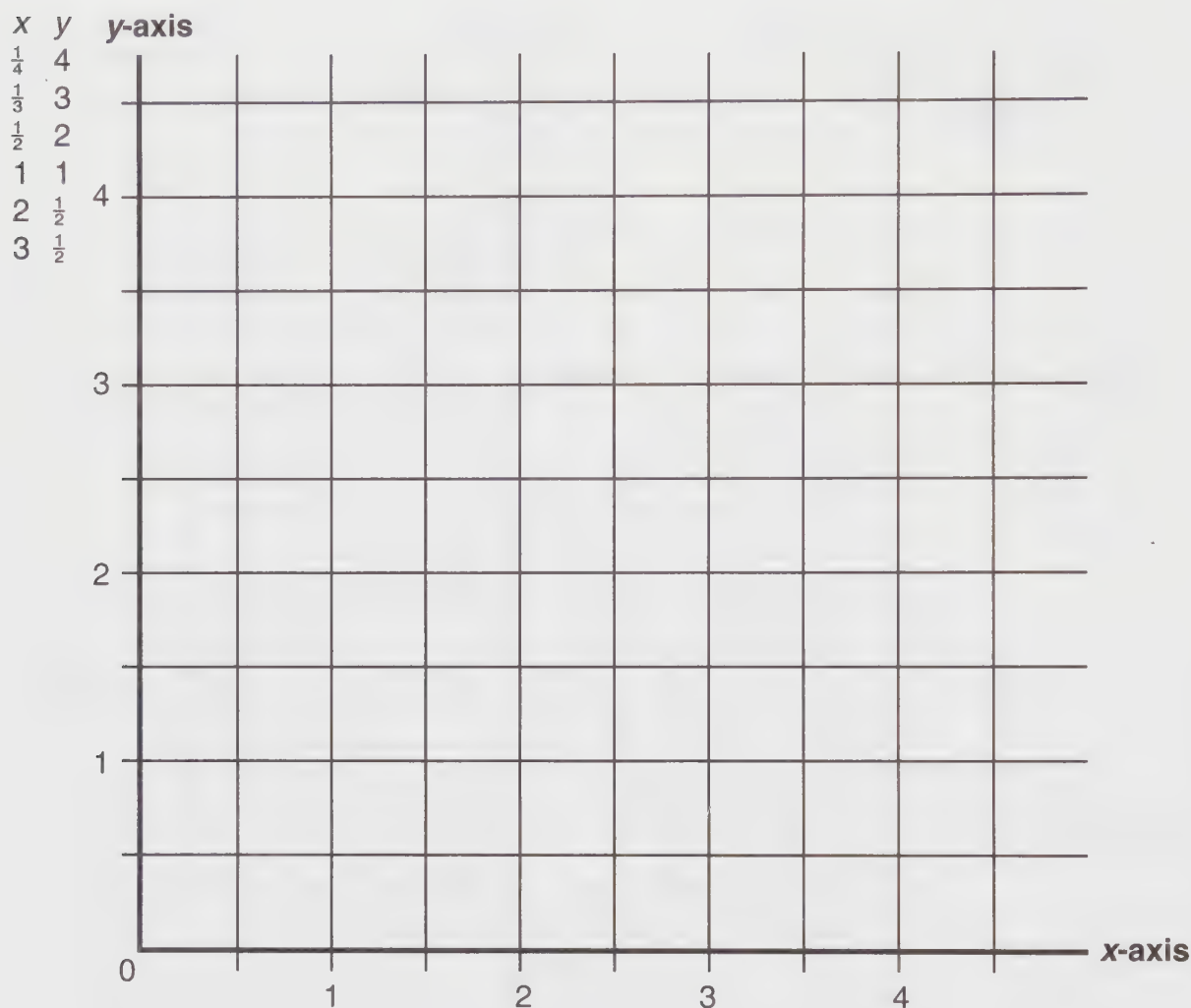
x	y
-4	16
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16



Notice that the portion of the graph to the left of the y -axis is a mirror image of the part that is to the right of the y -axis. It is like the y -axis is a mirror, and the graph to the left is a reflection of the graph to the right. We say that such a graph is symmetric with respect to the y -axis.

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2. Another type of equation is one that shows a **hyperbolic** relationship between y and x . In the simplest hyperbolic equation, the right-hand side of the equation is some number times $1/x$. A very simple hyperbolic equation is $y = 1/x$. A data table for this equation for $x = \frac{1}{4}$ to $x = 4$ is shown below. On the axes provided, plot a graph of this equation.

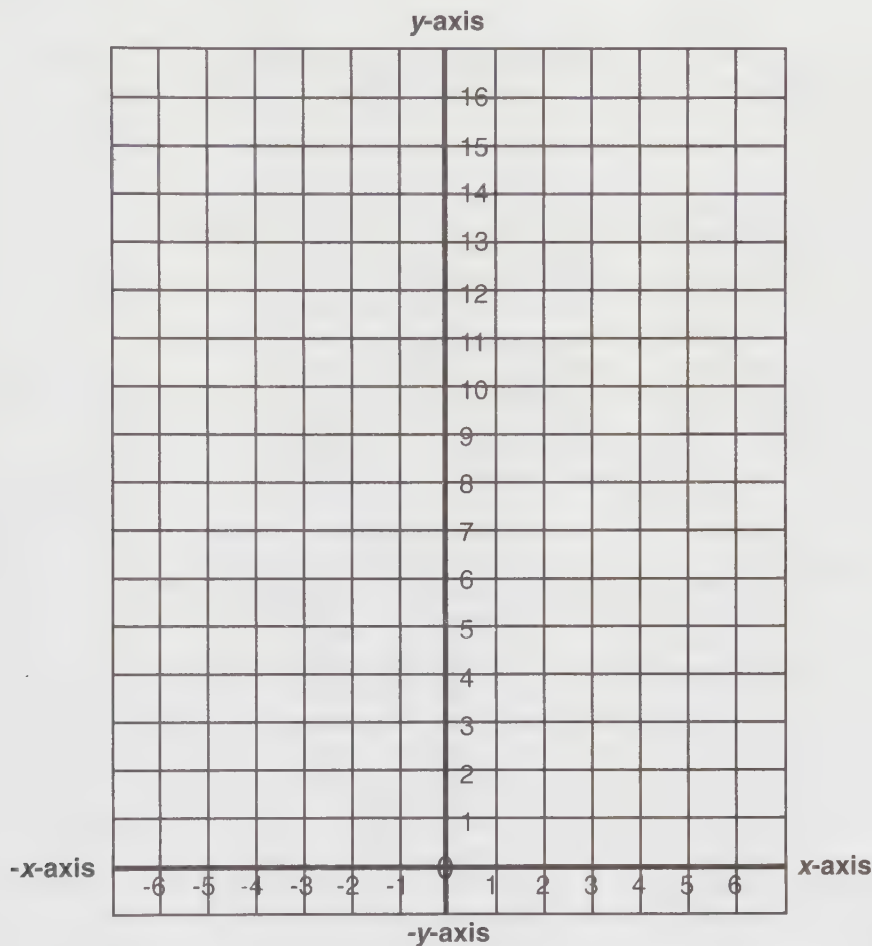


Notice that the graph gets closer and closer to the y -axis as x gets closer and closer to 0. Also notice that the graph gets closer and closer to the x -axis as x gets larger and larger (we sometimes say “as x gets indefinitely large” or “as x approaches infinity.”) On graphs such as this one, the x -axis and y -axis are asymptotes. Asymptotes are lines that the graph gets closer and closer to as x gets closer and closer to some value or as x gets indefinitely large.

Name _____ Date _____

3. Yet another type of equation is one that shows an **exponential** relationship between y and x . In an exponential equation, the variable x appears as the power of some number. A very simple form of an exponential equation is $y = 2^x$. A data table for this equation for $x = -3$ to $x = 4$ is shown below. On the axes provided, plot a graph of this equation.

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16



Notice that the graph approaches 0 as x becomes large and negative, and that it produces very large y values as x gets more and more positive. This behavior is typical for exponential equations. The x -axis is an asymptote for this graph.

ANSWER KEYS

Lesson One:

Exercise 1 (page 3)

1. \$32
2. \$32
3. \$45
4. \$107
5. 5th place
6. 4th place
7. 3rd place

Exercise 2 (page 4)

1. 11th floor
2. 17th floor
3. 58th floor
4. 47th floor

Exercise 3 (page 5)

1. 29
2. 13
3. 24
4. 51

Exercise 4 (page 6)

1. 35°F
2. 31°F
3. 31°F
4. 22°F

Exercise 5 (pages 7–8)

1. No
2. Yes
3. No
4. No
5. No
6. Yes
7. Yes
8. Yes
9. No

Exercise 6 (pages 9–11)

1. (a) +4, (b) -2, (c) Angie, (d) 36, (e) 40, (f) 34
2. (a) +2, (b) -2, (c) Jose, (d) 38, (e) 40, (f) 36

Exercise 7 (page 12)

1. (a) Par = 37, (b) Sam = 36, Sue = 35, Mel = 37, Meg = 41 (c) Sue (d) Sam = -1, Sue = -2, Mel = 0 (par), Meg = 4 (e) No

Exercise 8 (page 13)

1. Fill the 21-liter jar and pour it into the barrel. Fill the eight-liter jar and pour it into the barrel. Pour enough water out of the barrel to fill the six-liter jar. There will be 23 liters of water left in the barrel.

2. Fill the 24-gallon jar with water and pour it into the barrel. Pour enough water out of the barrel to fill the four-gallon jar. There will now be 20 gallons of water in the barrel

Lesson Two:

Exercise 1 (pages 14–15)

1. $10 + 5 = 15$; yes
2. $10 \times 8 = 80$; yes
3. (a) 72 (b) 480 (c) 7200
4. (a) 28° ; yes (b) -15° ; yes (c) 5° ; yes

Exercise 2 (page 16)

1. (a) 4; yes (b) no, no
2. (a) 19; yes (b) no; no

Exercise 3 (pages 17–18)

1. (a) 48° ; yes (b) -6° ; yes (c) -17° ; yes
2. (a) 62 ft., (b) 9,153 ft., (c) 9,215 ft., (d) 14,777 ft., (e) 14,715 ft., (f) 2,517 ft., (g) 102 ft., (h) yes

Exercise 4 (pages 19–20)

1. (a) 25 boxes; 3 rabbits (b) 17 boxes; 6 eggs (c) 20 boxes; 20 ornaments
2. 9 \$1 bills, 3 quarters
3. 4 \$5 bills, 19 quarters 4 \$1 bills, 3 quarters
4. (a) Tom Bus #4 (b) Meg Bus #2 (c) Jack Bus #4 (d) Jose Bus #1 (e) Lucia Bus #0 (f) Mike Bus #1 (g) Sally Bus #2 (h) Lupi Bus #0 (i) Jim Bus #3

Exercise 5 (pages 21–22)

1. $12; 5 + 7 = 12; 7 + 5 = 12$
2. Yes
3. Yes
4. Yes
5. Yes; (a) $10 \times 5 = 50; 5 \times 10 = 50$; (b) $8 \times 6 = 48; 6 \times 8 = 48$; (c) $125 \times 75 = 9,375; 75 \times 125 = 9,375$
6. Teacher check. For example: $5 - 3 = 2; 3 - 5 = -2$
7. Teacher check. For example: $15 \div 5 = 3; 5 \div 15 = \frac{1}{3}$

Exercise 6 (pages 23–24)

1. $(5 + 7) + 3 = 15$;
 $(7 + 3) + 5 = 15$;
 $(5 + 3) + 7 = 15$
2. $(4 + 2) + 3 = 9$;
 $(2 + 3) + 4 = 9$;
 $(4 + 3) + 2 = 9$
3. $5 + (2 + (3 + 10)) = 20$ for example
4. $(12 \times 5) \times 24 = 1440$;
 $(12 \times 24) \times 5 = 1440$;
 $12 \times (5 \times 24) = 1440$
5. $(3 \times 5) \times 4 = 60$;
 $(3 \times 4) \times 5 = 60$;
 $3 \times (5 \times 4) = 60$

Exercise 7 (page 25)

1. (a) $(12 \times 8) \times 6 = 576 \text{ in.}^3$;
 $(12 \times 6) \times 8 = 576 \text{ in.}^3$;
 $12 \times (8 \times 6) = 576 \text{ in.}^3$ (b) $(110 \times 75) \times 20 = 165,000 \text{ in.}^3$;
 $(110 \times 20) \times 75 = 165,000 \text{ in.}^3$;
 $110 \times (75 \times 20) = 165,000 \text{ in.}^3$

Exercise 8 (pages 26–28)

1. $(7 + 5) \times 6 = 72$;
 $(7 \times 6) + (5 \times 6) = 72$; yes
2. $(2 + 5) \times 52 = 364$;
 $(2 \times 52) + (5 \times 52) = 364$; yes
3. $(30 + 4) \times 11 = 374$;
 $(30 \times 11) + (4 \times 11) = 374$
4. (a) $20 \times 55 = 20 \times (50 + 5)$
 $= (20 \times 50) + (20 \times 5)$
 $= 1,100$
(b) $19 \times 110 = 19 \times (100 + 10)$
 $= (19 \times 100) + (19 \times 10)$
 $= 2,090$
(c) $14 \times 202 = 14 \times (200 + 2)$
 $= (14 \times 200) + (14 \times 2)$
 $= 2,828$
5. $(24 + 18) \times \frac{1}{3} = 14$;
 $24 \div 3 + 18 \div 3 = 14$;
or $(24 + 18) \div 3 = 14$
6. $(100 - 28) \div 3 = 24$
7. $2 \times (20 - 6) = \$28$
8. (a) $8 \times 79 = 8 \times (80 - 1) = 632$
(b) $19 \times 99 = 19 \times (100 - 1) = 1,881$
(c) $5 \times 208 = 5 \times (210 - 2) = 1,040$

Exercise 9 (pages 29–31)

1. (a) 10, (b) 1, (c) -11, (d) 138, (e) 1, (f) 9, (g) 25, (h) 18, (i) 43, (j) 8, (k) 33, (l) 45, (m) 5, (n) 10
2. $\frac{1}{3} \times 27 + \frac{1}{2} \times 24 + \frac{1}{4} \times 32 = 29$

ANSWER KEYS

- $\frac{1}{2} \times (8 + 12 + 6) = 13$;
 $\frac{1}{2} \times 8 + \frac{1}{2} \times 12 + \frac{1}{2} \times 6 = 13$
- $10 \times 31 + 5 \times 17 + 9 \times (-3) = 368$
- Eight shots outside third ring;
score = $10 \times 29 + 8 \times 21 + 6 \times 10 + 4 \times 7 + 8 \times (-2) = 530$

Lesson Three:

Exercise 1 (pages 32–34)

- (a) 27, (b) 16, (c) 32, (d) 125, (e) 243, (f) 1,296, (g) 1, (h) 9, (i) 1, (j) 13
- (a) 4, (b) 9, (c) 16, (d) 49, (e) 81, (f) 121
- (a) 8, (b) 27, (c) 64, (d) 125, (e) 512, (f) 1,728
- (a) 25 sq. in., (b) 81 sq. in., (c) 196 sq. in., (d) 13,225 sq. in., (e) 121 sq. in.
- (a) 8 cu. in., (b) 125 cu. in., (c) 512 cu. in., (d) 3,375 cu. in., (e) 1,157,625 cu. in., (f) 8,000,000 cu. in., (g) 343 cu. in.
- 125 spelling words
- 20,736 cans

Exercise 2 (pages 35–37)

- (a) 64, (b) 128, (c) 256
- (a) 2, (b) 4, (c) 8, (d) 16
- (a) 2, (b) 4, (c) 8, (d) 16, (e) 32, (f) 128, (g) 512, (h) 2,048
- (a) 2 cu. in., (b) 4 cu. in., (c) 16 cu. in., (d) 4,096 cu. in., (e) 16,777,216 cu. in.
- (a) \$2, (b) \$4, (c) \$8, (d) \$32, (e) \$1,024, (f) \$262,144

Exercise 3 (pages 38–40)

- (a) 3,200, (b) 6,400, (c) 25,600, (d) 102,400
- (a) 50, (b) 100, (c) 200, (d) 400
- (a) 10, (b) 20, (c) 40, (d) 80, (e) 160, (f) 640, (g) 2,560, (h) 10,240
- (a) 1,000 cu. in., (b) 2,000 cu. in., (c) 8,000 cu. in., (d) 2,048,000 (e) 8,388,608,000 cu. in.
- (a) \$200, (b) \$400, (c) \$800, (d) \$3,200, (e) \$102,400, (f) \$26,214,400

- (a) \$200, (b) \$400, (c) \$800, (d) \$1,600, (e) \$3,200, (f) \$6,400

Exercise 4 (page 41)

- (a) 400, (b) 1,600, (c) 6,400, (d) 409,600
- (a) \$1,024,000, (b) 0
- (a) 12 million, (b) 36 million, (c) 324 million

Exercise 5 (pages 42–43)

- 125
- 456,976
- 81
- (a) 1,000; largest = 999, smallest = 000
(b) 10,000; largest = 9999, smallest = 0000

Exercise 6 (pages 44–45)

- (a) 60, (b) 247, (c) 13, (d) 450
- (a) 8; largest = $111_2 = 7$, smallest = 000
(b) 16; largest = $1111_2 = 15$; smallest = 0000
- (a) 3, (b) 6, (c) 7, (d) 11

Exercise 7 (pages 46–48)

- (a) $\frac{1}{4}$, (b) $\frac{1}{9}$, (c) $\frac{1}{36}$, (d) $\frac{1}{5}$, (e) $\frac{1}{81}$, (f) $\frac{1}{64}$
- (a) 3 sq. in., (b) 24×2^{-3}
- (a) 32 sq. ft., (b) yes, (c) 1 sq. ft., (d) 32×2^{-5} (e) Teacher check
- (a) $\frac{1}{4} = 2^{-2}$, (b) $\frac{1}{8} = 2^{-3}$, (c) $\frac{1}{16} = 2^{-4}$, (d) $\frac{1}{32} = 2^{-5}$, (e) $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{32} = \frac{1}{32}$
- (a) 8,100 acres cut; 16,200 acres left
(b) 5,400 acres cut; 10,800 left
(c) 3,600 acres cut; 7,200 left
- (a) $\frac{1}{8}$, (b) $\frac{1}{16}$, (c) $\frac{1}{32}$

Lesson Four:

Exercise 1 (page 49)

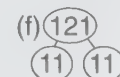
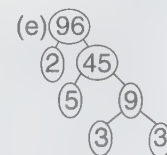
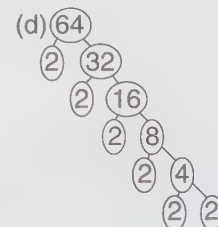
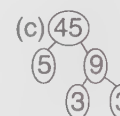
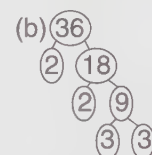
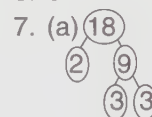
- 1 row - 36 chairs,
2 rows - 18 chairs,
3 rows - 12 chairs,
4 rows - 9 chairs,
6 rows - 6 chairs,
9 rows - 4 chairs,
18 rows - 2 chairs,
36 rows - one chair
- 1 column - 48 rows,
2 columns - 24 rows,
3 columns - 16 rows,
4 columns - 12 rows,

6 columns - 8 rows,
12 columns - 4 rows,
16 columns - 3 rows,
24 columns - 2 rows,
48 columns - 1 row

- 40 teams 1 person/team,
20 teams 2/team,
10 teams - 4/team,
8 teams 5/team,
5 teams - 8/team,
4 teams 10/team,
2 teams - 20/team,
1 team 40/team

Exercise 2 (pages 50–51)

- (a) prime, (b) not prime, (c) not prime, (d) prime, (e) not prime, (f) not prime, (g) not prime, (h) not prime, (i) not prime
- 17 and 19, 41 and 43, 59 and 61, 71 and 73
- 60
- 22,680
- 13
- 62



ANSWER KEYS

Exercise 3 (page 52)

- (a) $3/2 = 1.5$; not evenly divisible by 2
(b) $4/2 = 2$; evenly divisible by 2
(c) $7/2 = 3.5$; not evenly divisible by 2
(d) $8/2 = 4$; evenly divisible by 2
(e) $2/2 = 1$; evenly divisible by 2
- (a) $(2 + 4 + 3)/3 = 3$; evenly divisible by 3
(b) $(1 + 8 + 7)/3 = 5.33...$; not evenly divisible by 3
(c) $(8 + 3 + 4)/3 = 5$; evenly divisible by 3
(d) $(9 + 5 + 1)/3 = 5$; evenly divisible by 3
(e) $(6 + 4 + 0 + 2)/3 = 4$; evenly divisible by 3
- (a) $62/4 = 15.5$; not evenly divisible by 4
(b) $12/4 = 3$; evenly divisible by 4
(c) $34/4 = 8.5$; not evenly divisible by 4
(d) $56/4 = 14$; evenly divisible by 4
(e) $02/4 = 0.5$; not evenly divisible by 4

Exercise 4 (page 53–55)

- (a) 6, (b) 3, (c) 8, (d) 15, (e) 7, (f) 16, (g) 4, (h) 9, (i) 16
- 6 people per group, 3 groups of boys, 4 groups of girls
- 8 items per bag, 8 bags of candy bars, 9 bags of lollipops
- 9 people per table, 6 tables for daughter's friends, 7 tables for son-in-law's friends, 13 total tables
- 4 students per hiking group, 3 groups of British students, 2 groups of French students, 4 groups of German students, 9 total hiking groups
- 5 cans per box, 9 boxes of peas, 10 boxes of beans, and 6 boxes of corn, 25 total boxes

Exercise 5 (pages 56–58)

- (a) 18, (b) 45, (c) 48, (d) 90, (e) 84, (f) 544, (g) 40, (h) 63, (i) 84
- 35 quarts

- 30 soldiers
- 55 miles
- 24 feet
- 105 bushels

Lesson Five:

Exercise 1 (page 61)

- (a) 6, (b) 9, (c) 7, (d) 3, (e) 8, (f) 4, (g) 5, (h) 3, (i) 5, (j) 3, (k) 5, (l) 7
- 121
- 169
- 125
- 729
- 81
- 16,807

Exercise 2 (pages 62–64)

- (a) $\sqrt{16} = 4$ in., (b) $\sqrt{49} = 7$ in., (c) $\sqrt{144} = 12$ in.
- (a) 3 inches, (b) 5 inches, (c) 13 inches
- 7
- 3 and 2
- (a) -8, (b) -9, (c) -4, (d) -3, (e) -2, (f) -3
- (a) 24, (b) 78, (c) 50, (d) $17\sqrt{5}$, (e) 42, (f) 36, (g) 319, (h) $4\sqrt{7}$

Exercise 3 (page 65)

- (a) 13 inches, (b) 20 inches, (c) 52 inches

Exercise 4 (pages 66–68)

- (a) 32, (b) 500, (c) 2,187, (d) 540, (e) 1,120, (f) 7,734,375, (g) 16, (h) 125, (i) 64, (j) 16, (k) 875, (l) 245, (m) 4,096, (n) 15,625, (o) 1,296
- Age = 28
- 81
- 1,000

Exercise 5 (pages 69–70)

- (a) 10, (b) 4, (c) $\sqrt{6}$, (d) 80, (e) 210, (f) 495, (g) 6, (h) 5, (i) $\sqrt[3]{3}$, (j) 4, (k) 14, (l) 10, (m) 9, (n) 5, (o) 8
- $7\sqrt{2}$ gallons
- $5\sqrt{2}$ gallons
- 6 sq.in.
- 7 cu.in
- $4\sqrt{6}$ ft.

Exercise 6 (pages 71–73)

- (a) 12.50, (b) 24.25, (c) 85.8, (d) 256.9, (e) 305.75, (f) 42.4
- (a) 25 and $\frac{1}{2}$, (b) 14 and $\frac{1}{4}$, (c) 55 and $\frac{3}{4}$, (d) 256 and $\frac{1}{8}$,

- (e) 305 and $\frac{3}{4}$, (f) 42 and $\frac{9}{10}$

- 2.5 times as long
- 7.75 inches
- 10.5 inches
- 7.25 inches taller
- 10 pints
- 38.25 sq. inches
- 9 hours 30 minutes
- 4 hours 24 minutes
- 6.75 hours

Exercise 7 (pages 74–76)

- (a) 2.7×10^2 , (b) 5.6×10^3 , (c) 9.87×10^5 , (d) 3.18×10^8
- (a) 250, (b) 5,700, (c) 922,000, (d) 123,000,000
- (a) 5.29×10^5 , (b) 1.7×10^7 , (c) 4.01×10^3 , (d) 6.73×10^4
- (a) 5.25×10^5 , (b) 1.14×10^6 , (c) 3.9×10^{16} , (d) 2.0×10^2 , (e) 1.2×10^4

Lesson Six:

Exercise 1 (pages 77–78)

- (a) 12, (b) 3, (c) 3, (d) 16
- (a) 20 sq. in., (b) 108 sq. in., (c) 400 sq. ft.

Exercise 2 (page 79)

- (a) 12 sq. in., (b) 6 sq. ft., (c) 77 sq. ft.
- (a) 12.57 sq. in., (b) 254.5 sq. in., (c) 78.54 sq. ft.

Exercise 3 (page 80)

- (a) 60 cu. in., (b) 864 cu. in., (c) 2,800 cu. ft.
- (a) 33.56 cu. in., (b) 268.1 cu. in.

Exercise 4 (pages 81–82)

- (a) 30 mph, (b) 20 ft./sec., (c) 2,200 mph
- (a) 120 miles, (b) 150 feet, (c) 6.0×10^4 miles
- (a) 3 hours, (b) 30 sec., (c) 500 sec.
- (a) 2 cu. in., (b) 5 cu. in., (c) 5 cu. in.

Exercise 5 (pages 83–84)

- (a) 2, (b) 720, (c) 40,320
- (a) \$127.63, (b) \$296.05, (c) \$2,653.30
- (a) 5 Gs, (b) 0.4 Gs,

Exercise 6 (pages 85–87)

- Teacher check students' work.
(a) 2, (b) 3, (c) 5, (d) 2, (e) 4, (f) 2, (g) 3, (h) 9, (i) -8, (j) -2, (k) 2, (l) -14

ANSWER KEYS

2. $x - 60 = 3$; $x = 63$ in.
3. $8 - x = 4$; $x = \$4$
4. $2x = 140$; $x = 70$ lbs.
5. $x = 6/3$; $x = 2$ cups
6. $3x - 4 = 110$, $x = 38$ lbs.
7. $1/2(x - 3) = 1/4$; $x = 6/2$
8. $0.25x = 4 - 0.5$; $x = 14$
9. $x = 1 + 2/10 \times 25$; $x = 6$

Lesson Seven:

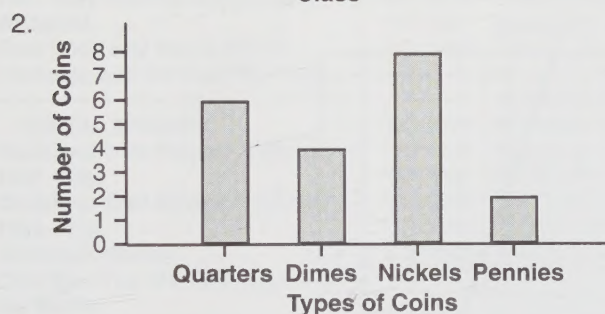
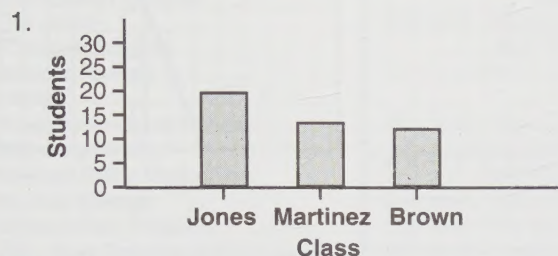
Exercise 1 (pages 88–89)

1–4. Teacher check number lines.

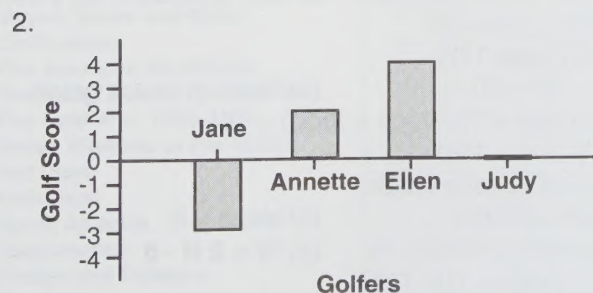
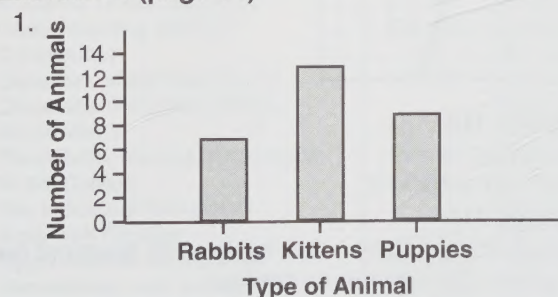
Exercise 2 (page 90)

1–4. Teacher check number lines.

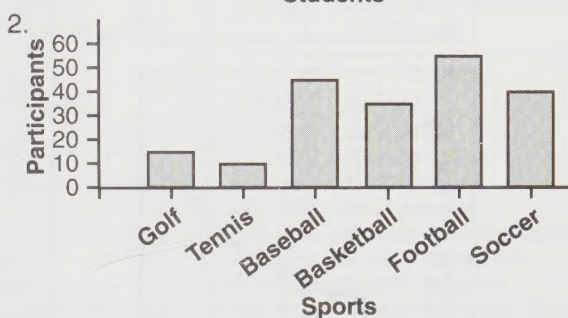
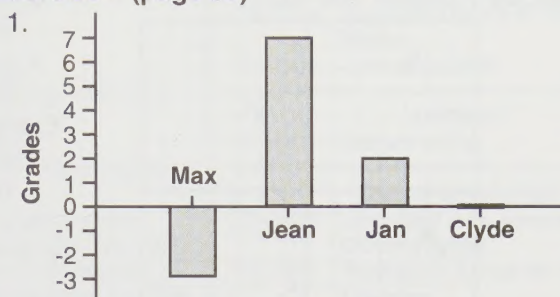
Exercise 3 (pages 91–93)



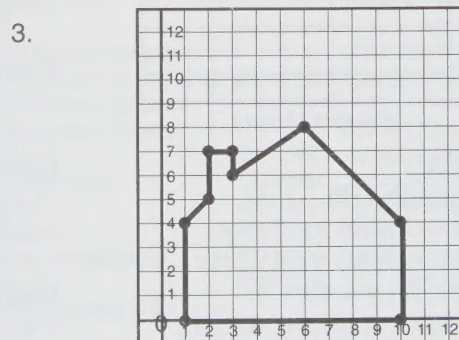
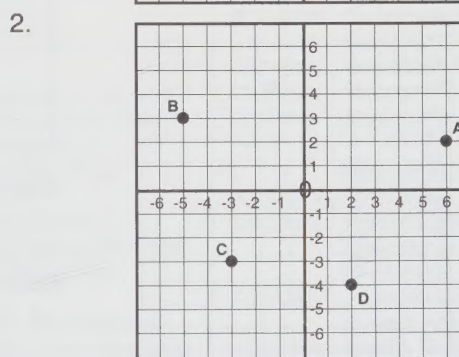
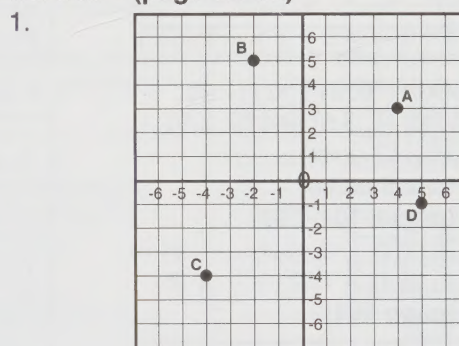
Exercise 4 (page 94)



Exercise 5 (page 95)



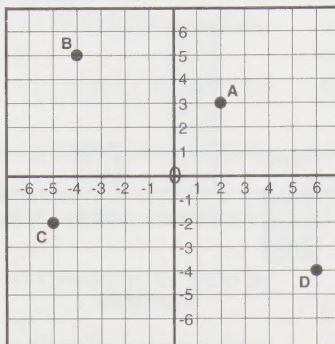
Exercise 6 (page 96–99)



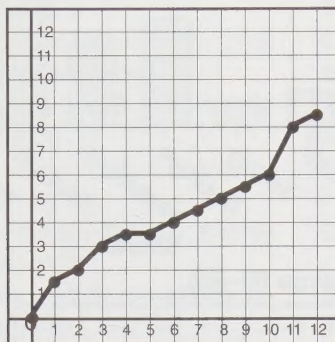
ANSWER KEYS

Exercise 7 (pages 100–101)

1.



2.



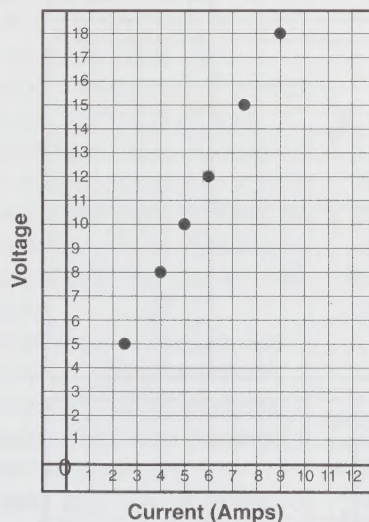
- (d) Between (10, 6) and (11, 8)
 (e) Between (4, 3.5) and (5, 3.5)
 (f) 1,200 ft. (12 units)
 (g) 850 ft. (8.5 units)

Exercise 8 (pages 102–108)

1. (1,2) 2. (2,1) 3. (5,4) 4. (4,4)
 5. (4,3) 6. (3,4) 7. 2 8. 2

Exercise 9 (page 104)

1.



Lesson Eight

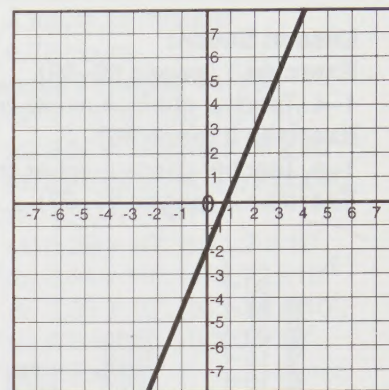
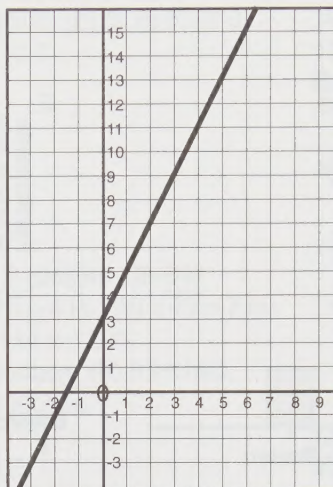
Exercise 1 (page 112)

1. (a) $x = 11 - 7y$, (b) $x = 2 - 4y$,
 (c) $x = 2y - 12$, (d) $x = 4 - 3/4y$
 2. (a) $y = 3x$, (b) $y = -9x + 4$,
 (c) $y = 1/2x + 6$, (d) $y = -3x + 7$

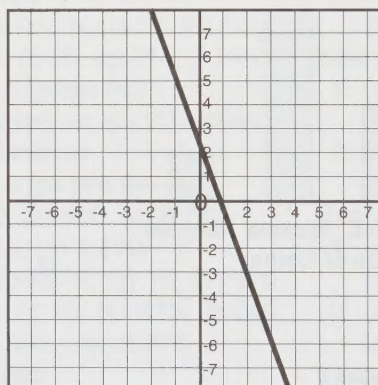
3. (a) x -int = 10, y -int = 10, (b) x -int = 10, y -int = 8,
 (c) x -int = -12, y -int = 9, (d) x -int = 10, y -int = $-35/2$
 4. (a) slope = 5, y -int = 11, (b) slope = 4, y -int = 8,
 (c) slope = $-4/5$, y -int = 4, (d) slope = $-3/4$, y -int = 0

Exercise 2 (pages 113–114)

1. (a) $y = 3x - 5$, (b) $y = -2x + 7$, (c) $y = 1/2x + 2$,
 (d) $y = -4x - 9$, (e) $y = -1/4x - 1$, (f) $y = 4$
 2. $y = 2x + 3$ 3. $2y - 5x + 4 = 0$



$$4. 3y + 12x = 18$$



Exercise 3 (page 115)

1. $y = 1/3x + 2$; $3y - x = 6$
 2. $y = -2x + 1$, $-2y + 4x = -2$

Exercise 4 (page 116)

1. (a) $y = 0.5x + 20$, (b) slope = 0.5; represents payment per mile, (c) y -int = 20; represents flat fee
 2. (a) $y = 5x + 300$, (b) slope = 5; reps. \$/student, (c) y -int = 300 reps. fixed cost

Exercise 5 (page 117)

1. (a) $d = 70t + 30$ (b) Teacher check graph.
 (c) intercept = 30; slope = $350/5 = 70$

Exercise 6 (page 118)

1. (a) Teacher check graph.
 (b) approximately (c) slope = 2
 (d) vertical intercept = -5 (e) $W = 2H - 5$

Exercise 7 (pages 119–121)

- 1–3. Teacher check graphs.

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